APPLIED MATHEMATICS COLLOQUIUM

Tom Hou
(Caltech)

Recent Progress on the Search of 3D Euler Singularities

Abstract: Whether the 3D incompressible Euler equations can develop a singularity in finite time from smooth initial data is one of the most challenging problems in mathematical fluid dynamics. This is closely related to the Clay Millennium Problem on 3D Navier-Stokes Equations. In this talk, we will present strong numerical evidence that the 3D Euler equations develop finite time singularities. To resolve the nearly singular solution, we develop specially designed adaptive (moving) meshes that are dynamically adjusted to the evolving solutions. With a maximum effective resolution of order 10^{12} in each dimension near the point of the singularity, we are able to advance the solution up to 10^{-6} distance from the predicted singularity time while maintaining a pointwise relative error of $O(10^{-4})$ in vorticity. We have applied all major blowup (non-blowup) criteria, including Beale-Kato-Majda, Constantin-Fefferman-Majda, and Deng-Hou-Yu to confirm the validity of the singularity. A careful local analysis also suggests that the blowing-up solution is highly anisotropic and is not of Leray type. However, the solution develops a self-similar structure near the point of the singularity in the radial and axial directions as the singularity time is approached. Finally, we prove rigorously the finite time blowup of a 1D model which captures the essential features of the finite time blowup of the 3D Euler equations.

Monday February 10, 2014
4:30 PM
Building E17, Room 122

Tea at 3:30-4:30PM in Building E17, Room 401A
(Math Dept. Common Room)

Applied Math Colloquium: http://www-math.mit.edu/amc/spring14/
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Massachusetts Institute of Technology
Department of Mathematics
Cambridge, MA 02139

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