APPLIED MATHEMATICS COLLOQUIUM

"Fast Spectral Function Approximation on Complex Geometries: Redundancy and Ill-conditioning are Sometimes Helpful"

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Abstract: It is challenging to construct a basis for a function space that is associated with a domain with complicated geometry, at least when one aims for spectral accuracy and fast transforms. A basis should be complete but not redundant, and this combination of properties is quite restrictive. Things become considerably easier by relaxing the latter property: it is nearly trivial to find complete but redundant sets, or frames, for pretty much any domain. This includes exotic ones like fractal sets. Unfortunately, redundancy implies non-uniqueness and this in turn may lead to ill-conditioning of several common operations.

In this talk we explore simple frames based on Fourier series for the approximation of functions on funny domains. These are very much related to Fourier extensions or Fourier continuations, but they are genuinely higher-dimensional. A number of results in this area seemingly defy common intuition about Fourier series. Perhaps most surprising is the observation that the approximation problem is unstable in theory, but numerically stable in practice. The stability is due, in fact, to extreme ill-conditioning.

We illustrate the results with a toolbox implemented in Julia. Being based on Fourier series, most operations can be carried out very efficiently using the FFT, in spite of the lack of periodicity or any sort of regularity of the domain under consideration.

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Applied Math Colloquium: <u>http://www-math.mit.edu/amc/fall14/</u> Math Department: <u>http://www-math.mit.edu</u>



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