APPLIED MATHEMATICS COLLOQUIUM

ON THE SOLVABILITY AND APPLICATION OF NUMERICAL METHODS TO CERTAIN FREDHOLM-LIKE INTEGRAL EQUATIONS OF ANTENNA THEORY

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ABSTRACT:

Straight, thin wires are often used as receiving or transmitting antennas and have been studied for many years. The basic unknown is the current along the antenna, which satisfies a one-dimensional, first-kind, Fredholm-like integral equation, usually referred to as Hallen's equation (HE). There are two choices of kernel, referred to as the "exact" and the "approximate" kernel. There are a number of ways of driving the antenna. The various combinations give rise to several versions of HE. The importance of HE is twofold: From the practical point of view, extensions of HE apply to many "real-life" antennas. From the educational point of view, students are often introduced to computational electromagnetics through the application of numerical methods to HE. The central results of this talk concern the difficulties associated with the application of numerical methods to HE.

The exact kernel is logarithmically singular; in contrast, the approximate kernel is differentiable. Using analytic-continuation arguments, we discuss how this implies that HE has no solution when the approximate kernel is used, something not mentioned in many recent textbooks. This rather peculiar situation motivates two questions: (a) What do we obtain when we apply a numerical method to HE? (b) When are numerical solutions obtained with the two kernels similar, and in what sense? We address these questions by applying Galerkin's method with pulse functions to the antenna of infinite length. Here, the infinite integration limits permit a detailed analytical study, in which Fourier transforms and asymptotic analysis play a key role. Once the infinite antenna is understood, the study of the finite antenna is greatly facilitated.

We further discuss an unexpected, exact relation between two versions of HE, other numerical methods, and the importance of round-off error. For the exact kernel, we use convergence acceleration techniques to improve upon Galerkin's method.

MONDAY, MAY 9, 2005
4:15 PM
Building 4, Room 231

Refreshments at 3:30 PM in Building 2, Room 349.

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