The Atiyah-Patodi-Singer index theorem

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To my mother, Isla Louise Melrose

PREFACE

This text is a somewhat expanded version of lecture notes written during, and directly after, a course at MIT in the Spring of 1991. Most of the participants had attended a course the preceding Fall on pseudodifferential operators on compact manifolds without boundary, including the Hodge theorem and the construction of the fundamental solution for the wave equation. Approximately this level of sophistication is assumed of the reader. The intention of the second course was to embed the Atiyah-Patodi-Singer index theorem in an analytic framework analogous to that provided by the theory of pseudodifferential operators for the Atiyah-Singer theorem. Since this treatment leads to a variety of current research topics it is presented here in the hope that it will be of use to a wider audience.

There are many people to thank. Foremost I am grateful to the members of the audience of the course for their tolerance and enthusiasm. I am especially grateful to Paolo Piazza for his comments during the course and also as a collaborator in work related to this subject. Others from whom I have learnt in this way are Xianzhe Dai, Charlie Epstein, Dan Freed, Rafe Mazzeo and Gerardo Mendoza. To the last of these I would like to take this opportunity to apologize for my part, whatever that was, in the somewhat mysterious non-appearance of the paper [64] on which a considerable part of Chapter 6, and indeed the general 'b-philosophy,' is based.

More generally I am happy to acknowledge the influence, through conversation, on my approach to this subject of Michael Atiyah, Ezra Getzler, Lars Hörmander, Werner Müller, Bob Seeley, Iz Singer and Michael Taylor. I am indebted to Antônio sá Barreto, Xianzhe Dai, Charlie Epstein and Maciej Zworski who tolerated my neglect of other projects during the process of writing, to Tanya Christiansen, Xianzhe Dai, Andrew Hassell, Lars Hörmander, Gerd Grubb, Rafe Mazzeo, Paolo Piazza and Lorenzo Ramero for comments on the manuscript and especially to Jillian Melrose for her forbearance. To Judy Romvos special thanks for turning my rather crude scrawlings into the original lecture notes.

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