MIT Integration Bee Qualifying Exam Answers **21 January 2025**

1 $\displaystyle\int \frac{x+\sqrt{x}}{1+\sqrt{x}}\,dx = \boxed{\frac{2}{3}x^{3/2}}$

2 $\displaystyle\int \frac{e^{x+1}}{e^x+1}dx = \boxed{e\log(e^x+1)}$

3 $\displaystyle\int \sqrt[3]{3\sin(x)-\sin(3x)}\,dx = \boxed{-\sqrt[3]{4}\cos(x)}$

4 $\displaystyle\int_1^{e^e} \frac{\log\left(x^{\log(x^x)}\right)}{x^2}\,dx = \boxed{\frac{e^3}{3}}$

5 $\displaystyle\int_{-\pi/2}^{\pi/2} \cos(20x)\sin(25x)\,dx = \boxed{0}$

6 $\displaystyle\int_0^{2\pi} \sin(x)\cos(x)\tan(x)\cot(x)\sec(x)\csc(x)\,dx = \boxed{2\pi}$

7 $\displaystyle\int \frac{x\log(x)\cos(x)-\sin(x)}{x\log^2(x)}\,dx = \boxed{\frac{\sin(x)}{\log(x)}}$

8 $\displaystyle\int_1^2 (2^{x-1}+\log_2(2x))\,dx = \boxed{3}$

9 $\displaystyle\int_0^1 x^{2024}(1-x^{2025})^{2025}\,dx = \boxed{\frac{1}{2025\cdot 2026}}$

10 $\displaystyle\int_0^{10} x\left(x-\frac{1}{2}\right)(x-1)\,dx = \boxed{2025}$

11 $\displaystyle\int_0^{20} \left\lceil \frac{\lfloor x \rfloor}{2} \right\rceil \, dx = \boxed{100}$

12 $\displaystyle\int \sqrt[3/1]{x \sqrt[4/2]{x \sqrt[5/3]{x \sqrt[6/4]{\dots}}}} \, dx = \boxed{\frac{x^2}{2}}$

13 $\displaystyle\int \frac{e^{2x}(x^2+x)}{(xe^x)^4+1} \, dx = \boxed{\frac{1}{2}\arctan(x^2e^{2x})}$

14 $\displaystyle\int \left(\sec^4(x) - \tan^4(x)\right) \, dx = \boxed{2\tan(x) - x}$

15 $\displaystyle\int_0^1 \sqrt{x(1-x)} \, dx = \boxed{\frac{\pi}{8}}$

16 $\displaystyle\int \frac{\sin(4x)\cos(x)}{\cos(2x)\sin(x)} \, dx = \boxed{2x + \sin(2x)}$

17 $\displaystyle\int \sin(x)\sinh(x) \, dx = \boxed{\frac{1}{2}(\sin(x)\cosh(x) - \cos(x)\sinh(x))}$

18 $\displaystyle\int_0^{\pi/3} \sin(x)\cos\left(\frac{\pi}{3} - x\right) \, dx = \boxed{\frac{\pi}{4\sqrt{3}}}$

19 $\displaystyle\int \left(\cos(x) + \cos\left(x + \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right)\right)^2 \, dx = \boxed{0}$

20 $\displaystyle\int_0^1 \left(\sum_{k=1}^{\infty} (-1)^k x^{2k}\right) \, dx = \boxed{\frac{\pi}{4} - 1}$