MIT Integration Bee Qualifying Exam Answers **18 January 2022**

1 $\displaystyle\int \frac{1+\cos x}{x+\sin x}\,dx = \boxed{\log(x+\sin x)}$

2 $\displaystyle\int_1^{\sqrt{3}} \frac{\arctan x + \operatorname{arccot} x}{x}\,dx = \boxed{\frac{\pi}{4}\log(3)}$

3 $\displaystyle\int_0^{2022} x^2 - \lfloor x \rfloor \lceil x \rceil\,dx = \boxed{674}$

4 $\displaystyle\int \frac{\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \boxed{-\frac{1}{2}x + \frac{1}{4}e^{2x}}$

5 $\displaystyle\int \frac{x}{\sqrt{x-1}+\sqrt{x+1}}\,dx = \boxed{\frac{(x+1)^{\frac{5}{2}}}{5} - \frac{(x+1)^{\frac{3}{2}}}{3} - \frac{(x-1)^{\frac{5}{2}}}{5} - \frac{(x-1)^{\frac{3}{2}}}{3}}$

6 $\displaystyle\int_0^{\pi} \cos(x+\cos x)\,dx = \boxed{0}$

7 $\displaystyle\int x^3 \sin(x^2)\,dx = \boxed{\frac{\sin(x^2) - x^2\cos(x^2)}{2}}$

8 $\displaystyle\int \frac{x}{1-x^4}\,dx = \boxed{\frac{1}{4}\log\left(\frac{1+x^2}{1-x^2}\right)}$

9 $\displaystyle\int \frac{1}{\cosh^2 x}\,dx = \boxed{\tanh(x)}$

10 $\displaystyle\int_0^1 e^{e^x} - e^{e^x - x}\,dx = \boxed{e^{e-1} - e}$

11 $\lim_{n\to\infty} \int_0^3 \underbrace{\sin\left(\frac{\pi}{3}\sin\left(\frac{\pi}{3}\sin\left(\ldots\sin\left(\frac{\pi}{3}x\right)\ldots\right)\right)\right)}_{n \text{ sin's}} dx = \boxed{\frac{3}{2}}$

12 $\int_0^1 \sqrt{1-\sqrt{x}}\, dx = \boxed{\frac{8}{15}}$

13 $\int \frac{x^3}{1+x+\frac{x^2}{2}+\frac{x^3}{6}}\, dx = \boxed{6\left(x - \log\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)\right)}$

14 $\int \sin(x+\sin(x)) - \sin(x-\sin(x))\, dx = \boxed{-2\cos(\sin(x))}$

15 $\int \tan^4(x)\sec^3(x) + \tan^2(x)\sec^5(x)\, dx = \boxed{\frac{1}{3}\tan^3(x)\sec^3(x)}$

16 $\int (1+\log x)\log\log x\, dx = \boxed{x\log x\log\log x - x}$

17 $\int \frac{1}{1+\sin x} + \frac{1}{1+\cos x} + \frac{1}{1+\tan x} + \frac{1}{1+\cot x} + \frac{1}{1+\sec x} + \frac{1}{1+\csc x}\, dx = \boxed{3x}$

18 $\int \frac{1}{\sqrt{x-x^2}}\, dx = \boxed{2\arcsin(\sqrt{x})}$

19 $\int_0^{\frac{1}{2}} \sum_{n=0}^{\infty} \binom{n+3}{n} x^n\, dx = \boxed{\frac{7}{3}}$

20 $\int \frac{1}{1+\cos^2 x}\, dx = \boxed{\frac{1}{\sqrt{2}}\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}$