This is the qualifying test for the 2015 Integration Bee. Finalists will be notified by email by midnight. You have 20 minutes to solve as many of the given 20 integrals as you can. Each integral is worth 1 point. In order to receive full credit you must express your answer in terms of x for indefinite integrals or simplified expressions in terms of constants for definite integrals, and **you must record your answers on the answer sheet**. There is no partial credit. The "log" symbol denotes the natural logarithm. In your answers, it is not necessary to include the arbitrary constant C nor the absolute value sign around the argument of a logarithm.

MIT Integration Bee Qualifying Exam **20 January 2015**

1 $\displaystyle\int (\cos^4 x - \sin^4 x)\, dx = \frac{1}{2}\sin 2x$

2 $\displaystyle\int \frac{x}{\sqrt{2+4x}}\, dx = \frac{1}{3}(x-1)\sqrt{x+\frac{1}{2}}$

3 $\displaystyle\int_0^8 \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx = 2\sin(2\sqrt{2})$

4 $\displaystyle\int \sec x\, dx = \operatorname{sech}^{-1} \cos x$

5 $\displaystyle\int_0^{\pi/2} \frac{e^{\sin x}}{\tan x \csc x}\, dx = e - 1$

6 $\displaystyle\int_1^e x(\log x)^2 dx = \frac{e^2}{4} - \frac{1}{4}$

7 $\displaystyle\int \frac{1}{5+4\sqrt{x}+x}\, dx$

$= \log(x+4\sqrt{x}+5) - 4\tan^{-1}(\sqrt{x}+2)$

8 $\displaystyle\int (2015)^x\, dx = \frac{(2015)^x}{\log(2015)}$

9 $\displaystyle\int_0^2 \frac{x}{(x-3)(x+5)^2}\, dx$

$\displaystyle= \frac{1}{28} - \frac{3}{64}\log\left(\frac{21}{5}\right)$

10 $\displaystyle\int \frac{\log(1+\log x)}{x}\, dx$

$= \log(1+\log x) + \log x(\log(1+\log x) - 1)$

11 $\displaystyle\int \sqrt{\csc x - \sin x}\, dx = 2^{3/4}$

12 $\displaystyle\int \frac{1}{\sqrt{x^2+25}}\, dx = \log|x+\sqrt{x^2+25}|$

13 $\displaystyle\int_2^e \frac{\log^2 x - 1}{x\log^2 x}\, dx = -\frac{(\log 2 - 1)^2}{\log 2}$

14 $\displaystyle\int e^{3x} \arctan e^x dx$

$$= \frac{e^{3x} \arctan(e^x)}{3} - \frac{e^{2x}}{6} + \frac{\log(1+e^{2x})}{6}$$

15 $\displaystyle\int_0^4 \frac{|x-1|}{|x-2|+|x-3|}$

$$= 2 + \frac{9}{4}\log 3 - \frac{3}{4}\log 5$$

16 $\displaystyle\int_0^{2\pi} \frac{1}{\sin^4 x + \cos^4 x}\, dx = 2\pi\sqrt{2}$

17 $\displaystyle\int \frac{1+e^x}{1-e^x}\, dx = x - 2\log|1-e^x|$

18 $\displaystyle\int \tan^4 x\, dx = \frac{\tan^3 x}{3} - \tan x + x$

19 $\displaystyle\int \sin x \tan^2 x\, dx = \cos x + \frac{1}{\cos x}$

20 $\displaystyle\int \frac{x+1}{x^2+2x+3}\, dx = \frac{1}{2}\log(x^2+2x+3)$