MIT Integration Bee: Semifinals

(Time limit per integral: 4 minutes)

Semifinal #1 Problem 1

$$\int_0^\infty \frac{\sqrt[3]{x}}{1+x^2}\,dx$$

Semifinal #1 Problem 1

$$\int_0^\infty \frac{\sqrt[3]{x}}{1+x^2}\,dx = \boxed{\frac{\pi}{\sqrt{3}}}$$

Semifinal #1 Problem 2

$$\int_{-\pi}^{\pi} \log\left(82+2\left(\cos(x)\sqrt{81-\sin^2(x)}-\sin^2(x)\right)\right)\,dx$$

Semifinal #1 Problem 2

$$\int_{-\pi}^{\pi} \log\left(82+2\left(\cos(x)\sqrt{81-\sin^2(x)}-\sin^2(x)\right)\right)\,dx$$

$$= \boxed{2\pi\log(80)}$$

Semifinal #1 Problem 3

$$\int (3x^2 + 7x - 5)\left(x + \frac{1}{x}\right) e^{x+\frac{1}{x}}\, dx$$

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$$\int (3x^2 + 7x - 5)\left(x + \frac{1}{x}\right) e^{x+\frac{1}{x}}\,dx$$

$$= \boxed{(3x^3 - 2x^2 + 5x)e^{x+\frac{1}{x}}}$$

Semifinal #1 Problem 4

$$\int_0^\infty \frac{x}{e^{2x}+1}\,dx$$

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$$\int_0^\infty \frac{x}{e^{2x}+1}\,dx = \boxed{\frac{\pi^2}{48}}$$

MIT Integration Bee: Semifinal Tiebreakers

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Semifinal Tiebreakers Problem 1

$$\int \frac{x+24}{x^3+25x^2+144x}\,dx$$

Semifinal Tiebreakers Problem 1

$$\int \frac{x+24}{x^3+25x^2+144x}\,dx$$

$$= \boxed{\frac{1}{6}\log(x) - \frac{5}{21}\log(x+9) + \frac{1}{14}\log(x+16)}$$

Semifinal #2 Problem 1

$$\int \frac{\sqrt{(x^6+1)(x^2+1)}}{x^3}\,dx$$

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$$\int \frac{\sqrt{(x^6+1)(x^2+1)}}{x^3}\,dx$$

$$= \boxed{\tfrac{1}{2}\left(\left(1-\tfrac{1}{x^2}\right)\sqrt{x^4-x^2+1}+\operatorname{arctanh}\left(\frac{x^2-1}{\sqrt{x^4-x^2+1}}\right)\right)}$$

Semifinal #2 Problem 2

$$\int_0^1 \frac{\log(x)}{\sqrt{x - x^2}}\,dx$$

Semifinal #2 Problem 2

$$\int_0^1 \frac{\log(x)}{\sqrt{x - x^2}}\, dx = \boxed{-2\pi \log(2)}$$

Semifinal #2 Problem 3

$$\int_1^{\infty} \left(\sum_{k=0}^{\infty} (-1)^k \max(0, x-k) \right)^{-2} dx$$

Semifinal #2 Problem 3

$$\int_1^\infty \left(\sum_{k=0}^{\infty} (-1)^k \max(0, x-k) \right)^{-2} dx = \boxed{1 + \frac{\pi^2}{6}}$$

Semifinal #2 Problem 4

$$\int_0^1 \left\lfloor \log_2 \left(x - 2^{\lfloor \log_2 x \rfloor} \right) \right\rfloor \, dx$$

Semifinal #2 Problem 4

$$\int_0^1 \left\lfloor \log_2 \left(x - 2^{\lfloor \log_2 x \rfloor} \right) \right\rfloor dx = \boxed{-4}$$