MIT Integration Bee: Regular Season

(Time limit per integral: 2 minutes)

Regular Season Problem 1

$$\int_0^\infty \frac{dx}{(x+1+2\sqrt{x})^2}$$

Regular Season Problem 1

$$\int_0^\infty \frac{dx}{(x+1+2\sqrt{x})^2} = \boxed{\frac{1}{3}}$$

Regular Season Problem 2

$$\int x^2 \cos(\operatorname{arccsc}(x))\, dx$$

Regular Season Problem 2

$$\int x^2 \cos(\operatorname{arccsc}(x))\, dx = \boxed{\frac{1}{3}(x^2-1)^{3/2}}$$

Regular Season Problem 3

$$\int_0^{1/2025} \left(\sum_{k=1}^{\infty} \frac{(2025x)^k e^{-2025x}}{k!} \right) dx$$

Regular Season Problem 3

$$\int_0^{1/2025} \left(\sum_{k=1}^{\infty} \frac{(2025x)^k e^{-2025x}}{k!} \right) dx = \boxed{\frac{1}{2025e}}$$

Regular Season Problem 4

$$\int \frac{dx}{1 - x^4}$$

Regular Season Problem 4

$$\int \frac{dx}{1-x^4} = \boxed{\frac{1}{2}\arctan(x) + \frac{1}{4}\log(1+x) - \frac{1}{4}\log(1-x)}$$

Regular Season Problem 5

$$\int_{-\pi/2}^{\pi/2} \cos(20x)\cos(25x)\,dx$$

Regular Season Problem 5

$$\int_{-\pi/2}^{\pi/2} \cos(20x)\cos(25x)\,dx = \boxed{\frac{2}{9}}$$

Regular Season Problem 6

$$\int_{-2}^{2} \max(x, x^2, x^3)\, dx$$

Regular Season Problem 6

$$\int_{-2}^{2} \max(x, x^2, x^3)\, dx = \boxed{\frac{83}{12}}$$

Regular Season Problem 7

$$\int_0^{2025} \left\{\sqrt{x}\right\}\, dx$$

Regular Season Problem 7

$$\int_0^{2025} \{\sqrt{x}\}\, dx = \boxed{1020}$$

Regular Season Problem 8

$$\int_0^{2\pi} \left| \sin(x) + \frac{1}{2} \right| \, dx$$

Regular Season Problem 8

$$\int_0^{2\pi} \left| \sin(x) + \frac{1}{2} \right| \, dx = \boxed{2\sqrt{3} + \frac{\pi}{3}}$$

Regular Season Problem 9

$$\int x\left(\frac{1}{2}+\log x\right)\log(\log x)\,dx$$

Regular Season Problem 9

$$\int x \left(\frac{1}{2} + \log x \right) \log(\log x) \, dx$$
$$= \boxed{\frac{1}{2} x^2 \log x \log(\log x) - \frac{1}{4} x^2}$$

Regular Season Problem 10

$$\int \frac{\cos^4(x) - 1}{\sin^8(x)} \, dx$$

Regular Season Problem 10

$$\int \frac{\cos^4(x) - 1}{\sin^8(x)}\, dx = \boxed{\frac{1}{5}\cot(x)(2\csc^4(x) + \csc^2(x) + 2)}$$

Regular Season Problem 11

$$\int \sqrt{x+\sqrt{x^2-1}}\,dx$$

Regular Season Problem 11

$$\int \sqrt{x+\sqrt{x^2-1}}\,dx = \boxed{\frac{\sqrt{2}}{3}\left((x-1)^{3/2}+(x+1)^{3/2}\right)}$$

Regular Season Problem 12

$$\int \frac{x^3 - x}{x^6 - 1}\, dx$$

Regular Season Problem 12

$$\int \frac{x^3 - x}{x^6 - 1}\, dx = \boxed{\frac{1}{\sqrt{3}} \arctan\left(\frac{2x^2 + 1}{\sqrt{3}}\right)}$$

Regular Season Problem 13

$$\int \sqrt{x^2 - 1}\, dx$$

Regular Season Problem 13

$$\int \sqrt{x^2-1}\,dx = \boxed{\frac{1}{2}\left(x\sqrt{x^2-1} - \log(x+\sqrt{x^2-1})\right)}$$

Regular Season Problem 14

$$\int \begin{pmatrix} \sin(x)\sin(\sin(x))\sin(\cos(x)) \\ \quad +\cos(x)\cos(\sin(x))\cos(\cos(x)) \end{pmatrix} dx$$

Regular Season Problem 14

$$\int \left(\begin{aligned} &\sin(x)\sin(\sin(x))\sin(\cos(x)) \\ &\quad + \cos(x)\cos(\sin(x))\cos(\cos(x)) \end{aligned} \right) dx$$
$$= \boxed{\sin(\sin(x))\cos(\cos(x))}$$

Regular Season Problem 15

$$\int \left(\frac{\log(x)}{x} \right)^2 \, dx$$

Regular Season Problem 15

$$\int \left(\frac{\log(x)}{x}\right)^2 dx = \boxed{-\frac{2+2\log(x)+\log^2(x)}{x}}$$

Regular Season Problem 16

$$\int \sin^2(2x)e^{2x}\,dx$$

Regular Season Problem 16

$$\int \sin^2(2x)e^{2x}\,dx$$
$$= \boxed{\left(\frac{1}{4} - \frac{1}{10}\sin(4x) - \frac{1}{20}\cos(4x)\right)e^{2x}}$$

Regular Season Problem 17

$$\int_0^{\frac{7}{2}} \sqrt{x + \frac{1}{\sqrt{x + \frac{1}{\sqrt{x + \cdots}}}}}\, dx$$

Regular Season Problem 17

$$\int_0^{\frac{7}{2}} \sqrt{x + \frac{1}{\sqrt{x + \frac{1}{\sqrt{x + \cdots}}}}}\, dx = \boxed{\frac{14}{3} + \log 2}$$

Regular Season Problem 18

$$\int_0^\infty (x+1)^4 e^{-x^2}\, dx$$

Regular Season Problem 18

$$\int_0^{\infty} (x+1)^4 e^{-x^2}\, dx = \boxed{4 + \frac{19}{8}\sqrt{\pi}}$$

Regular Season Problem 19

$$\int_0^{\pi/2} \cos(3x)\cos(5x)\cos(7x)\,dx$$

Regular Season Problem 19

$$\int_0^{\pi/2} \cos(3x)\cos(5x)\cos(7x)\,dx = \boxed{\frac{14}{45}}$$

Regular Season Problem 20

$$\int_{-1}^{1} e^{2x} \sin(\sinh x)\, dx$$

Regular Season Problem 20

$$\int_{-1}^{1} e^{2x} \sin(\sinh x)\, dx$$
$$= \boxed{4(\sin(\sinh 1) - \sinh 1 \cos(\sinh 1))}$$