MIT Integration Bee: Finals

(Time limit per integral: 5 minutes)

Finals Problem 1

$$\int \tan(x)\sqrt{2+\sqrt{4+\cos(x)}}\,dx$$

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$$\int \tan(x)\sqrt{2+\sqrt{4+\cos(x)}}\,dx$$

$$= \boxed{-4\sqrt{2+\sqrt{4+\cos(x)}}-2\log\left(\frac{\sqrt{2+\sqrt{4+\cos(x)}}-2}{\sqrt{2+\sqrt{4+\cos(x)}}+2}\right)}$$

Finals Problem 2

$$\int_0^\infty \frac{dx}{(x+1+\lfloor 2\sqrt{x} \rfloor)^2}$$

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$$\int_0^\infty \frac{dx}{(x+1+\lfloor 2\sqrt{x} \rfloor)^2} = \boxed{\frac{2\pi^2}{3} - \frac{73}{12}}$$

Finals Problem 3

$$\int_0^{10} \left\lfloor \left(\frac{1+\sqrt{5}}{2} \right)^{\lfloor x \rfloor} \right\rfloor dx$$

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$$\int_0^{10} \left\lfloor \left(\frac{1+\sqrt{5}}{2} \right)^{\lfloor x \rfloor} \right\rfloor dx = \boxed{193}$$

Finals Problem 4

$$\int_0^{\pi} \max(|2\sin(x)|,|2\cos(2x)-1|)^2 \cdot \min(|\sin(2x)|,|\cos(3x)|)^2 \, dx$$

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$$\int_0^{\pi} \max(|2\sin(x)|,|2\cos(2x)-1|)^2 \cdot \min(|\sin(2x)|,|\cos(3x)|)^2 \, dx$$

$$= \boxed{\pi}$$

Finals Problem 5

$$\int_0^1 \left(\sqrt{\frac{1}{4x^2} + \frac{1}{x} - x} - \sqrt{\frac{x^4}{4} - x + 1} - \frac{1}{2x} \right) \, dx$$

Finals Problem 5

$$\int_0^1 \left(\sqrt{\frac{1}{4x^2} + \frac{1}{x} - x} - \sqrt{\frac{x^4}{4} - x + 1} - \frac{1}{2x} \right) \, dx = \boxed{-\frac{1}{6}}$$