MIT Integration Bee: Semifinal #1

(Time limit per integral: 4 minutes)

Semifinal #1 Problem 1

$$\int_{-\infty}^{\infty} \frac{(x^3 - 4x)\sin x + (3x^2 - 4)\cos x}{(x^3 - 4x)^2 + \cos^2 x}\,dx$$

Semifinal #1 Problem 1

$$\int_{-\infty}^{\infty} \frac{(x^3-4x)\sin x + (3x^2-4)\cos x}{(x^3-4x)^2+\cos^2 x}\,dx = \boxed{-3\pi}$$

Semifinal #1 Problem 2

$$\int_0^\infty \frac{xe^{-2x}}{e^{-x}+1}\,dx$$

Semifinal #1 Problem 2

$$\int_0^\infty \frac{xe^{-2x}}{e^{-x}+1}\,dx = \boxed{1-\frac{\pi^2}{12}}$$

Semifinal #1 Problem 3

$$\int_0^{\pi/2} \sin(\cot^2(x)) \sec^2(x)\, dx$$

Semifinal #1 Problem 3

$$\int_0^{\pi/2} \sin(\cot^2(x)) \sec^2(x)\, dx = \boxed{\sqrt{\frac{\pi}{2}}}$$

Semifinal #1 Problem 4

$$\int \cosh^2(3x)\tanh(2x)\,dx$$

Semifinal #1 Problem 4

$$\int \cosh^2(3x)\tanh(2x)\,dx$$

$$= \boxed{-\frac{1}{2}\cosh(2x) + \frac{1}{12}\cosh(6x) + \frac{1}{4}\log(\cosh(2x))}$$

MIT Integration Bee: Semifinal Tiebreakers

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Semifinal Tiebreakers Problem 1

$$\int \sec^5(x)\,dx$$

Semifinal Tiebreakers Problem 1

$$\int \sec^5(x)\, dx$$

$$= \boxed{\tfrac{1}{4}\sec^3(x)\tan(x)+\tfrac{3}{8}\sec(x)\tan(x)+\tfrac{3}{8}\log(\sec(x)+\tan(x))}$$

Semifinal Tiebreakers Problem 2

$$\int_{-\infty}^{\infty} \operatorname{sech}\left(2x+1-\frac{1}{x-1}-\frac{2}{x+1}\right)\,dx$$

Semifinal Tiebreakers Problem 2

$$\int_{-\infty}^{\infty} \operatorname{sech}\left(2x + 1 - \frac{1}{x-1} - \frac{2}{x+1}\right) dx = \boxed{\frac{\pi}{2}}$$

MIT Integration Bee: Semifinal #2

(Time limit per integral: 4 minutes)

Semifinal #2 Problem 1

$$\int_0^\infty \frac{\sin(x)\sin(2x)\sin(3x)}{x^3}\,dx$$

Semifinal #2 Problem 1

$$\int_0^\infty \frac{\sin(x)\sin(2x)\sin(3x)}{x^3}\,dx = \boxed{\pi}$$

Semifinal #2 Problem 2

$$\int (1 + \log x)(1 + \log \log x)\, dx$$

Semifinal #2 Problem 2

$$\int (1 + \log x)(1 + \log\log x)\, dx$$
$$= \boxed{-x + x\log x + x\log x \log\log x}$$

Semifinal #2 Problem 3

$$\int_0^\infty \frac{e^{-x^2}}{\left(x^2+\frac{1}{2}\right)^2}\,dx$$

Semifinal #2 Problem 3

$$\int_0^\infty \frac{e^{-x^2}}{\left(x^2+\frac{1}{2}\right)^2}\,dx = \boxed{\sqrt{\pi}}$$

Semifinal #2 Problem 4

$$\int \tan x \sec^2 x \cos(2x) e^{2\cos x}\, dx$$

Semifinal #2 Problem 4

$$\int \tan x \sec^2 x \cos(2x) e^{2\cos x}\, dx$$

$$= \boxed{-\left(\sec x + \frac{\sec^2 x}{2}\right) e^{2\cos x}}$$