

MIT Integration Bee: Semifinal #1

(Time limit per integral: 3 minutes)

Semifinal #1 Problem 1

$$\int e^{\cos x} \cos(2x + \sin x) dx$$

Semifinal #1 Problem 1

$$\begin{aligned} & \int e^{\cos x} \cos(2x + \sin x) dx \\ &= \boxed{e^{\cos x}(\sin(x + \sin x) - \sin(\sin x))} \end{aligned}$$

Semifinal #1 Problem 2

$$\int_0^1 (9x^9 - x^{90} + 9x^{99} - x^{900} + 9x^{909} - x^{990} + 9x^{999} - x^{9000} + \dots) dx$$

Semifinal #1 Problem 2

$$\int_0^1 (9x^9 - x^{90} + 9x^{99} - x^{900} + 9x^{909} - x^{990} + 9x^{999} - x^{9000} + \dots) dx = \boxed{1}$$

Semifinal #1 Problem 3

$$\int_0^{\pi} \frac{2\cos(x) - \cos(2021x) - 2\cos(2022x) - \cos(2023x) + 2}{1 - \cos(2x)} dx$$

Semifinal #1 Problem 3

$$\int_0^{\pi} \frac{2\cos(x) - \cos(2021x) - 2\cos(2022x) - \cos(2023x) + 2}{1 - \cos(2x)} dx$$
$$= \boxed{2022\pi}$$

Semifinal #1 Problem 4

$$\int \frac{3 \log x - 1 + 2x}{x \log x + x^2 + 2x^4} dx$$

Semifinal #1 Problem 4

$$\int \frac{3 \log x - 1 + 2x}{x \log x + x^2 + 2x^4} dx = \boxed{3 \log x - \log(\log x + x + 2x^3)}$$

MIT Integration Bee: Semifinal #2

(Time limit per integral: 3 minutes)

Semifinal #2 Problem 1

$$\int (\sqrt{x+1} - \sqrt{x})^\pi dx$$

Semifinal #2 Problem 1

$$\int (\sqrt{x+1} - \sqrt{x})^\pi dx \\ = \frac{1}{2} \left(\frac{(\sqrt{x+1} - \sqrt{x})^{\pi+2}}{\pi+2} - \frac{(\sqrt{x+1} - \sqrt{x})^{\pi-2}}{\pi-2} \right)$$

Semifinal #2 Problem 2

$$\int_{-2}^2 (((((x^2 - 2)^2 - 2)^2 - 2)^2 - 2) \, dx$$

Semifinal #2 Problem 2

$$\int_{-2}^2 (((((x^2 - 2)^2 - 2)^2 - 2)^2 - 2) \, dx = \boxed{-\frac{8}{255}}$$

Semifinal #2 Problem 3

$$\int_0^\infty \frac{\tanh(x)}{x \cosh(2x)} dx$$

Semifinal #2 Problem 3

$$\int_0^\infty \frac{\tanh(x)}{x \cosh(2x)} dx = \boxed{\log 2}$$

Semifinal #2 Problem 4

$$\int \sin(4 \arctan(x)) dx$$

Semifinal #2 Problem 4

$$\int \sin(4 \arctan(x)) dx = \boxed{-\frac{4}{1+x^2} - 2 \log(1+x^2)}$$