MIT Integration Bee: Regular Season

(Time limit per integral: 2 minutes)

Regular Season Problem 1

$$\int_0^{2\pi} \max(\sin(x), \sin(2x))\, dx$$

Regular Season Problem 1

$$\int_0^{2\pi} \max(\sin(x), \sin(2x))\, dx = \boxed{\frac{5}{2}}$$

Regular Season Problem 2

$$\int_0^1 \frac{\frac{x+1}{x+2}}{\frac{x+3}{x+4}}\,dx$$

Regular Season Problem 2

$$\int_0^1 \frac{\frac{x+1}{x+2}}{\frac{x+3}{x+4}}\,dx = \boxed{1 - \log\left(\frac{81}{64}\right)}$$

Regular Season Problem 3

$$\int_0^3 \left(\min\left(2x, \frac{5-x}{2}\right) - \max\left(-\frac{x}{2}, 2x-5\right)\right)\, dx$$

Regular Season Problem 3

$$\int_0^3 \left(\min\left(2x, \frac{5-x}{2}\right) - \max\left(-\frac{x}{2}, 2x-5\right) \right) dx = \boxed{5}$$

Regular Season Problem 4

$$\int \underbrace{1 - \frac{1}{1 - \frac{1}{\ddots \frac{1}{1-\frac{1}{x}}}}}_{2023\ (1-)\text{'s}}\, dx$$

Regular Season Problem 4

$$\int \underbrace{1-\frac{1}{1-\frac{1}{\ddots\frac{1}{1-\frac{1}{x}}}}}_{2023\,(1-)\text{'s}}\,dx = \boxed{x-\log x}$$

Regular Season Problem 5

$$\int_0^{\pi/2} x \cot x \, dx$$

Regular Season Problem 5

$$\int_0^{\pi/2} x \cot x \, dx = \boxed{\frac{\pi}{2} \log 2}$$

Regular Season Problem 6

$$\int \left(\frac{x^6 + x^4 - x^2 - 1}{x^4} \right) e^{x+1/x}\, dx$$

Regular Season Problem 6

$$\int \left(\frac{x^6 + x^4 - x^2 - 1}{x^4} \right) e^{x+1/x} \, dx$$

$$= \boxed{\left(x^2 - 2x + 4 - \frac{2}{x} + \frac{1}{x^2} \right) e^{x+1/x}}$$

Regular Season Problem 7

$$\int \frac{dx}{\sqrt{(x+1)^3(x-1)}}$$

Regular Season Problem 7

$$\int \frac{dx}{\sqrt{(x+1)^3(x-1)}} = \boxed{\sqrt{\frac{x-1}{x+1}}}$$

Regular Season Problem 8

$$\int_0^{\pi} x \sin^4(x)\, dx$$

Regular Season Problem 8

$$\int_0^{\pi} x \sin^4(x)\, dx = \boxed{\frac{3\pi^2}{16}}$$

Regular Season Problem 9

$$\int \binom{x}{5}^{-1} dx$$

Regular Season Problem 9

$$\int \binom{x}{5}^{-1} dx$$

$$= \boxed{5\log(x)-20\log(x-1)+30\log(x-2)-20\log(x-3)+5\log(x-4)}$$

Regular Season Problem 10

$$\int \frac{\sin(2x)\cos(3x)}{\sin^2 x \cos^3 x}\,dx$$

Regular Season Problem 10

$$\int \frac{\sin(2x)\cos(3x)}{\sin^2 x \cos^3 x}\, dx = \boxed{8 \log \sin x - 6 \log \tan x}$$

Regular Season Problem 11

$$\int \left(\sqrt{2\log x} + \frac{1}{\sqrt{2\log x}} \right) \, dx$$

Regular Season Problem 11

$$\int \left(\sqrt{2\log x} + \frac{1}{\sqrt{2\log x}} \right) dx = \boxed{x\sqrt{2\log x}}$$

Regular Season Problem 12

$$\int \frac{\log(\cos(x))}{\cos^2(x)}\,dx$$

Regular Season Problem 12

$$\int \frac{\log(\cos(x))}{\cos^2(x)}\,dx = \boxed{\tan(x)\log(\cos(x)) + \tan(x) - x}$$

Regular Season Problem 13

$$\int_0^{\frac{\pi}{2}+1} \sin(x - \sin(x - \sin(x - \cdots))) \, dx$$

Regular Season Problem 13

$$\int_0^{\frac{\pi}{2}+1} \sin(x - \sin(x - \sin(x - \cdots))) \, dx = \boxed{\frac{3}{2}}$$

Regular Season Problem 14

$$\int_0^{100} \lfloor x \rfloor x \lceil x \rceil \, dx$$

Regular Season Problem 14

$$\int_0^{100} \lfloor x \rfloor x \lceil x \rceil \, dx = \boxed{\frac{100^4 - 100^2}{4} = 24997500}$$

Regular Season Problem 15

$$\int_{-\infty}^{\infty} \frac{\frac{1}{(x-1)^2} + \frac{3}{(x-3)^4} + \frac{5}{(x-5)^6}}{1 + \left(\frac{1}{x-1} + \frac{1}{(x-3)^3} + \frac{1}{(x-5)^5}\right)^2}\, dx$$

Regular Season Problem 15

$$\int_{-\infty}^{\infty} \frac{\frac{1}{(x-1)^2} + \frac{3}{(x-3)^4} + \frac{5}{(x-5)^6}}{1 + \left(\frac{1}{x-1} + \frac{1}{(x-3)^3} + \frac{1}{(x-5)^5}\right)^2}\, dx = \boxed{3\pi}$$

Regular Season Problem 16

$$\int_0^{\pi} \sin^2(3x + \cos^4(5x))\, dx$$

Regular Season Problem 16

$$\int_0^{\pi} \sin^2(3x + \cos^4(5x))\, dx = \boxed{\frac{\pi}{2}}$$

Regular Season Problem 17

$$\int_0^5 (-1)^{\lfloor x \rfloor + \lfloor x/\sqrt{2} \rfloor + \lfloor x/\sqrt{3} \rfloor}\, dx$$

Regular Season Problem 17

$$\int_0^5 (-1)^{\lfloor x \rfloor + \lfloor x/\sqrt{2} \rfloor + \lfloor x/\sqrt{3} \rfloor}\, dx = \boxed{8\sqrt{2} + 6\sqrt{3} - 21}$$

Regular Season Problem 18

$$\int_0^\infty \frac{x+1}{x+2} \cdot \frac{x+3}{x+4} \cdot \frac{x+5}{x+6} \cdot \cdots \, dx$$

Regular Season Problem 18

$$\int_0^\infty \frac{x+1}{x+2} \cdot \frac{x+3}{x+4} \cdot \frac{x+5}{x+6} \cdot \cdots \, dx = \boxed{0}$$

Regular Season Problem 19

$$\int_0^{\frac{\pi}{2}} \frac{\sin(23x)}{\sin(x)}\,dx$$

Regular Season Problem 19

$$\int_0^{\frac{\pi}{2}} \frac{\sin(23x)}{\sin(x)}\,dx = \boxed{\frac{\pi}{2}}$$

Regular Season Problem 20

$$\int_1^{100} \left(\frac{\lfloor x/2 \rfloor}{\lfloor x \rfloor} + \frac{\lceil x/2 \rceil}{\lceil x \rceil} \right) dx$$

Regular Season Problem 20

$$\int_1^{100} \left(\frac{\lfloor x/2 \rfloor}{\lfloor x \rfloor} + \frac{\lceil x/2 \rceil}{\lceil x \rceil} \right) dx = \boxed{\frac{197}{2}}$$