

MIT Integration Bee: Semifinal #1
(Time limit per integral: 3 minutes)

Semifinal #1 Problem 1

$$\int_0^{\infty} \frac{x(e^{-x} + 1)}{e^x - 1} dx$$

Semifinal #1 Problem 1

$$\int_0^{\infty} \frac{x(e^{-x} + 1)}{e^x - 1} dx = \boxed{\frac{\pi^2}{3} - 1}$$

Semifinal #1 Problem 2

$$\int_0^{\infty} x^5 e^{-x} \sin x \, dx$$

Semifinal #1 Problem 2

$$\int_0^{\infty} x^5 e^{-x} \sin x \, dx = \boxed{-15}$$

Semifinal #1 Problem 3

$$\int_{\frac{1}{2}}^2 \log \left(\frac{\log(x + \frac{1}{x})}{\log(x^2 - x + \frac{17}{4})} \right) dx$$

Semifinal #1 Problem 3

$$\int_{\frac{1}{2}}^2 \log \left(\frac{\log(x + \frac{1}{x})}{\log(x^2 - x + \frac{17}{4})} \right) dx = \boxed{-\frac{3}{2} \log 2}$$

Semifinal #1 Problem 4

$$\int_2^{\frac{5}{2}} \frac{(x^3 - 3x)^3 - 3(x^3 - 3x)}{\sqrt{x^2 - 4}} dx$$

Semifinal #1 Problem 4

$$\int_2^{\frac{5}{2}} \frac{(x^3 - 3x)^3 - 3(x^3 - 3x)}{\sqrt{x^2 - 4}} dx = \boxed{\frac{2^9 - 2^{-9}}{9}}$$

MIT Integration Bee: Semifinal # 2
(Time limit per integral: 3 minutes)

Semifinal # 2 Problem 1

$$\int_{-1}^1 \left| \left| \left| \left| \left| x \right| - \frac{2}{3} \right| - \frac{2}{3^2} \right| - \frac{2}{3^3} \right| - \dots \right| dx$$

Semifinal # 2 Problem 1

$$\int_{-1}^1 \left| \left| \left| \left| \left| x \right| - \frac{2}{3} \right| - \frac{2}{3^2} \right| - \frac{2}{3^3} \right| - \dots \right| dx = \boxed{\frac{1}{7}}$$

Semifinal # 2 Problem 2

$$\int \frac{1}{(x^2 + 1)^3} dx$$

Semifinal # 2 Problem 2

$$\int \frac{1}{(x^2 + 1)^3} dx = \frac{3}{8} \arctan(x) + \frac{3x^3 + 5x}{8(x^2 + 1)^2}$$

Semifinal # 2 Problem 3

$$\int_{-\infty}^{\infty} \frac{1}{x^2 - 2x \cot(x) + \csc^2(x)} dx$$

Semifinal # 2 Problem 3

$$\int_{-\infty}^{\infty} \frac{1}{x^2 - 2x \cot(x) + \csc^2(x)} dx = \boxed{\pi}$$

Semifinal # 2 Problem 4

$$\int_0^{\frac{\pi}{6}} \log(\sqrt{3} + \tan x) dx$$

Semifinal # 2 Problem 4

$$\int_0^{\frac{\pi}{6}} \log(\sqrt{3} + \tan x) dx = \boxed{\frac{\pi \log(2)}{6}}$$