MIT Integration Bee: Finals

(Time limit per integral: 4 minutes)

Finals Problem 1

$$\int \sqrt{(\sin(20x)+3\sin(21x)+\sin(22x))^2+(\cos(20x)+3\cos(21x)+\cos(22x))^2}\,dx$$

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$$\int \sqrt{(\sin(20x)+3\sin(21x)+\sin(22x))^2+(\cos(20x)+3\cos(21x)+\cos(22x))^2}\,dx$$

$$= \boxed{3x + 2\sin x}$$

Finals Problem 2

$$\int_0^\infty \frac{e^{-2x}\sin(3x)}{x}\,dx$$

Finals Problem 2

$$\int_0^\infty \frac{e^{-2x}\sin(3x)}{x}\,dx = \boxed{\arctan\frac{3}{2}}$$

Finals Problem 3

$$\int_0^{2\pi} \cos(2022x) \frac{\sin(10050x)}{\sin(50x)} \frac{\sin(10251x)}{\sin(51x)} \, dx$$

Finals Problem 3

$$\int_0^{2\pi} \cos(2022x) \frac{\sin(10050x)}{\sin(50x)} \frac{\sin(10251x)}{\sin(51x)} \, dx = \boxed{6\pi}$$

Finals Problem 4

$$\int_0^1 x^{\frac{1}{3}}(1-x)^{\frac{2}{3}}\,dx$$

Finals Problem 4

$$\int_0^1 x^{\frac{1}{3}}(1-x)^{\frac{2}{3}}\,dx = \boxed{\frac{2\pi}{9\sqrt{3}}}$$

Finals Problem 5

$$\left\lfloor \log_{10} \int_{2022}^{\infty} 10^{-x^3} \, dx \right\rfloor$$

Finals Problem 5

$$\left\lfloor \log_{10} \int_{2022}^{\infty} 10^{-x^3}\, dx \right\rfloor = \boxed{-2022^3 - 8}$$