18.440 Practice Midterm: 50 minutes, 100 points
Carefully and clearly show your work on each problem (without writing anything that is technically not true) and put a box around each of your final computations.

1. (30 points) Twenty people in a room each have independently random birthdays among 365 possibilities. Let P be the number of pairs of people that share a birthday (i.e., the number of ways of choosing a pair of two people that share a birthday). Let T be the number ways of choosing a triple of three people that share a birthday. (If everyone has the same birthday, then $P = 20 * 19/2$ and $T = 20 * 19 * 18/6$.) Compute the following:

(a) $\mathbb{E}[P]$

(b) $\mathrm{Var}(P)$

(c) $\mathbb{E}[T]$

(d) The probability that $P = 5$ and $T = 1$.

(e) The probability that $P = 5$ and $T = 0$.

(f) The probability that $P = 5$ and $T => 1$.

2. (20 points) Compute how many:

(a) Quadruples (w, x, y, z) of non-negative integers with $w + x + y + z = 50$.

(b) Ways to divide 15 books into five groups of size 1, 2, 3, 4, and 5.

(c) "Two pair" poker hands: (i.e. 2 cards of one denomination, 2 of another distinct denomination, and one of a third distinct denomination).

3. (20 points)

(a) Roll three dice. Find the probability that there are at least two sixes given that there is at least one six.

(b) Find the conditional probability that a standard poker hand has at least 3 aces given that it has at least 2.

4. (10 points) Suppose that the sample space S contains three elements $\{1, 2, 3\}$, with probabilities .5, .2, and .3 respectively. Suppose $X(s) = s^2 - 4$ for $s \in S$. Compute

(a) $\mathbb{E}X$.

(b) $\mathrm{Cov}(X, |X|)$.

5. (20 points) Suppose X is Poissonian random variable with parameter $\lambda_1 = 1$, Y is an independent Poissonian random variable with $\lambda_2 = 2$, and Z is a Poissonian random variable with parameter $\lambda_3 = 3$. Assume X and Y and Z are independent and compute the following:

(a) $P\{X + Y + Z = 8\}$

(b) $\mathrm{Cov}(X + 2Y, 2Y + 3Z)$

(c) $\mathbb{E}[XYZ]$

(d) $\mathbb{E}[X^2 Y^2 Z]$