Math 54-1 Your name: Key

Quiz 5, July 13, 2010

Please write your name **on each sheet**. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (4 pt) Is the set

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 1 \end{bmatrix} \right\}$$

a basis of $\mathbb{R}^3$?

Form the matrix with these columns:

$$A = \begin{bmatrix} 1 & 3 & 7 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 - 2R_2} \begin{bmatrix} \boxed{1} & 3 & 7 \\ 0 & \boxed{1} & 2 \\ 0 & 0 & \boxed{-3} \end{bmatrix}$$ is invertible (has 3 pivots).

Therefore, the set above is a basis of $\mathbb{R}^3$.

2. (6 pt) Consider the matrix $A$, which is row equivalent to $B$:

$$A = \begin{bmatrix} 1 & 5 & 2 & 10 \\ 1 & 5 & 0 & 4 \\ 0 & 0 & 2 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} \boxed{1} & 5 & 0 & 4 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Pivot columns: $1^{st}$ and $3^{rd}$

(a) Find a basis for Col $A$ and a basis for Nul $A$.

(b) Find a basis for Col $B$ and a basis for Nul $B$.

(a) Basis ~~for Col~~ $A$ – pivot columns of $A$:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \right\}$$

Basis for Nul $A$: $A\vec{x} = 0$ means

$$\vec{x} = x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -4 \\ 0 \\ -3 \\ 1 \end{bmatrix}, \; x_2, x_4 \in \mathbb{R} \Leftarrow \begin{cases} x_1 = -5x_2 - 4x_4 \\ x_2 \text{ free} \\ x_3 = -3x_4 \\ x_4 \text{ free} \end{cases}$$

$\downarrow$

Basis for Nul $A$ is $\left\{ \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$.

(b) ~~Basis A is row equiv~~

$B$ has $1^{st}$ and $3^{rd}$ pivot columns $\rightarrow$

$\rightarrow$ Basis for Col $B$: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$.

$A$ is row equivalent to $B$ $\rightarrow$ Nul $B$ = Nul $A$ $\rightarrow$ basis for Nul $B$ is the same as for Nul $A$.