Math 54-1, H/W 1

1.1. (3) The point of intersection satisfies two linear equations:

$$x_1 + 5x_2 = 7$$
$$x_1 - 2x_2 = -2$$

Augmented matrix: $\begin{bmatrix} 1 & 5 & 7 \\ 1 & -2 & -2 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{bmatrix} 1 & 5 & 7 \\ 0 & -7 & -9 \end{bmatrix} \longrightarrow$

$\xrightarrow{R_2 \leftarrow -\frac{1}{7}R_2} \begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & 9/7 \end{bmatrix} \xrightarrow{R_1 \leftarrow R_1 - 5R_2} \begin{bmatrix} \boxed{1} & 0 & 4/7 \\ 0 & \boxed{1} & 9/7 \end{bmatrix}.$

So, $x_1 = 4/7$, $x_2 = 9/7$, the intersection point is $(4/7, 9/7)$

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(5) We are already in REF. We use the third row to eliminate the elements above its pivot position:

$R_1 \leftarrow R_1 - 5R_3$, $R_2 \leftarrow R_2 + 3R_3$.

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(17) We need to solve the system
$$x_1 - 4x_2 = 1$$
$$2x_1 - x_2 = -3$$
$$-x_1 - 3x_2 = 4$$
; then,

the augmented matrix $\begin{bmatrix} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{bmatrix}$ is row reduced to $\begin{bmatrix} \boxed{1} & -4 & 1 \\ 0 & \boxed{7} & -5 \\ 0 & 0 & 0 \end{bmatrix}$.

The system is consistent → the 3 lines have a common pt.

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(24) (a) True (b) False; row equivalence means that one matrix can be transformed to the other by row operations (c) False; an inconsistent system has no solutions (d) True, by definition.

1.2 (15) (a) Consistent (no pivot in last column)
Unique (pivots in each of variable columns)
(b) Inconsistent (pivot in last column)

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(16) (a) Consistent, Unique (b) Consistent, Not unique (x_2 is free)

1.3 ⑥ $x_1\begin{bmatrix}-2\\3\end{bmatrix}+x_2\begin{bmatrix}8\\5\end{bmatrix}+x_3\begin{bmatrix}1\\-6\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$$\begin{bmatrix}-2x_1\\3x_1\end{bmatrix}+\begin{bmatrix}8x_2\\5x_2\end{bmatrix}+\begin{bmatrix}x_3\\-6x_3\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}-2x_1+8x_2+x_3\\3x_1+5x_2-6x_3\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$-2x_1+8x_2+x_3=0$$
$$3x_1+5x_2-6x_3=0$$

⑯ For example,

$0\cdot\vec{v}_1+0\cdot\vec{v}_2=\begin{bmatrix}0\\0\\0\end{bmatrix}$, $1\cdot\vec{v}_1+0\,\vec{v}_2=\begin{bmatrix}3\\0\\2\end{bmatrix}$, $0\cdot\vec{v}_1+1\cdot\vec{v}_2=\begin{bmatrix}-2\\0\\3\end{bmatrix}$,

$1\cdot\vec{v}_1+1\cdot\vec{v}_2=\begin{bmatrix}1\\0\\5\end{bmatrix}$, $-1\cdot\vec{v}_1-1\cdot\vec{v}_2=\begin{bmatrix}-1\\0\\-5\end{bmatrix}$.

⑳ Span $\{\vec{v}_1,\vec{v}_2\}$ is a plane through the origin, since neither of $\vec{v}_1, \vec{v}_2$ is a multiple of the other: