

Math 1B, Section 105
Quiz 8, October 28, 2009

Your name: Key

Please write your name on each sheet. Show your work clearly and in order, including the intermediate steps in the solutions and the final answer.

1. (4 pt) Approximate the function $f(x) = 1/x$ near the point $a = 1$ using the Taylor polynomial T_1 . Estimate the error of approximation on the interval $[1/2, 3/2]$ using Taylor's inequality. The answer should be a number independent of x .

$$f(x) = \frac{1}{x}, \quad f'(x) = -\frac{1}{x^2}, \quad f''(x) = \frac{2}{x^3}$$

$$a=1, \quad f(a) = 1, \quad f'(a) = -1$$

$$T_1(x) = 1 - 1(x-1) = \boxed{2-x}$$

$$\text{Taylor's inequality: } |f(x) - T_1(x)| \leq \frac{M|x-1|^2}{2},$$

where $M \geq |f''(t)|$ for $t \in [1/2, 3/2]$;

$|f''(t)| = \frac{2}{t^3}$ is decreasing \rightarrow can take $M = |f''(1/2)| =$

$$= \frac{2}{(1/2)^3} = 16.$$

Finally, if $x \in [1/2, 3/2]$, then $|x-1| \leq 1/2 \rightarrow |x-1|^2 \leq 1/4$

$$\text{So: for } x \in [1/2, 3/2], \quad \boxed{|f(x) - T_1(x)| \leq 2}$$

2. (6 pt) (a) Find the first nonzero term (the nonzero term with the least power of x) of the Maclaurin series for the function

$$f(x) = \frac{e^x}{1+x} - 1.$$

You may use the formula $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

- (b) Find an integer $k \geq 0$ such that the limit $\lim_{x \rightarrow 0} \frac{f(x)}{x^k} = c$ is a finite nonzero number. Evaluate c .

$$\textcircled{a} \quad \frac{1}{1+x} = 1 - x + x^2 + \dots$$

$$e^x = 1 + x + \frac{x^2}{2} + \dots$$

$$f(x) = \left(1 + x + \frac{x^2}{2} + \dots\right) (1 - x + x^2 + \dots) - 1 =$$

$$= 1 + x - x + \frac{x^2}{2} - x^2 + x^2 + \dots - 1 =$$

$$= \frac{x^2}{2} + \dots, \text{ where } (\dots) \text{ has powers } \geq 3 \text{ of } x.$$

First nonzero term is $\boxed{\frac{x^2}{2}}$.

\textcircled{b} Take $k=2$, then

$$\frac{f(x)}{x^2} = \frac{1}{2} + (\dots), \text{ where } (\dots) \text{ has powers } \geq 1 \text{ of } x.$$

As $x \rightarrow 0$, $(\dots) \rightarrow 0$; so,

$$\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = \frac{1}{2}$$

$$\boxed{\begin{matrix} k=2 \\ c = \frac{1}{2} \end{matrix}}$$