# Math 1B worksheet

Nov 2–4, 2009

In this worksheet:
**DE** means ‘differential equation’,
**IVP** means ‘initial value problem’.

1–4. Find general formulas for the solutions to the following DE and solve the following IVP:

$$\begin{cases} y' = 2xy^2, \\ y(0) = 1/2; \end{cases} \tag{1}$$

$$\begin{cases} y' = 2y, \\ y(1) = 7; \end{cases} \tag{2}$$

$$\begin{cases} (1+\cos x)y' = (1+e^{-y})\sin x, \\ y(0) = 0; \end{cases} \tag{3}$$

$$\begin{cases} y' = 3x^2 e^y, \\ y(0) = 1. \end{cases} \tag{4}$$

5–6. Find the curve passing through the given point and orthogonal to all curves in the given family:

$$y = e^{kx}, (1,2); \tag{5}$$

$$y = ke^x, (1,1). \tag{6}$$

7–8. Prove that the following power series solves the given DE:

$$y(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{1\cdot 3\cdot 5\cdots(2n+1)},\ y' = 1 + xy; \tag{7}$$

$$y(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^n n!},\ y' = -xy; \tag{8}$$

9. Sketch the direction fields for the following ODE and sketch the graph $y = y(x)$, where $y$ solves the given IVP:

$$y' = 1 - xy,\ y(0) = 0. \tag{9}$$

10. If $y$ solves the IVP from problem 1, find $y(1)$ approximately using Euler's method with step 1/2.

11. A tank contains 10 L of pure water. Brine that contains 0.05 kg of salt per liter of water enters the tank at a rate of 5 L/min. The solution is kept thoroughly mixed and drains from the tank at the same time. How much salt is in the tank after one hour?