(1) Do last year’s midterm and understand all solutions.
(2) Do the optional midterm review set.
(3) Come to office hours, Wed 6-8 PM (2-172, probably moving to 2-290).
(4) Know all the homework problems and all the solutions. Be able to look at any problem on the homework or in the book, or any similar problem, and know immediately how to do it.
(5) Outline of possible subjects for the midterm:
   • Eigenspaces and eigenvalues
   • Upper triangular and block upper triangular form matrices $M(T)$ for $T \in L(V)$
   • Eigenbases and diagonal matrices
   • Inner product spaces; norms; the parallelogram identity
   • Gram-Schmidt orthogonalization and block upper-triangular matrices in orthonormal bases
   • Orthogonal complements and projections
   • Adjoint operators and matrices in orthonormal bases
   • Self-adjoint, anti-self-adjoint and normal operators
   • The spectral theorems
   • Positive operators and matrices
   • Isometries and unitary matrices
   • Singular value decomposition and be able to do it (cf. PS9, #7)
   • Polar decomposition and be able to do it (cf. PS9, #7)
   • Characteristic polynomial for $2 \times 2$ matrices and for upper-triangular matrices
   • Trace and determinant (in terms of sum and product of eigenvalues, or as formulas, see PS10, #10)
   • Generalized eigenspaces (their definition, the fact that $V(\lambda) = \text{null}((T - \lambda I)^m)$ for large enough $m \geq 1$)
   • The decomposition theorem (in the strengthened form in class, and in the version of Theorem 8.23.(a) in the complex case)
   • The Cayley-Hamilton theorem for upper-triangular matrices and for transformations that admit an upper triangular matrix (in the latter case, that $\prod_{\lambda}(T - \lambda I)^{\dim V(\lambda)} = 0$)
   • Minimal polynomials (PS 10 #8)
(6) Definitions (there is a lot of repetition with the above):
   • Eigenspaces and eigenvalues
   • Trace, determinant, and characteristic polynomial of $2 \times 2$-matrices
   • A block upper-triangular (or block diagonal) matrix
   • Inner product spaces
   • Norm associated to an inner product space
   • Abstract normed spaces
   • The parallelogram identity
   • Orthogonal complement
   • Projection and orthogonal projection
• Adjoint operator
• Self-adjoint, anti-self-adjoint, and normal operators
• Orthonormal lists, orthonormal bases, and orthonormal eigenbases
• Singular values and a singular value decomposition
• Characteristic values of a complex operator or matrix in terms of upper-triangular form
• Trace and determinant in terms of eigenvalues
• Trace and determinant in terms of matrix coefficients (PS 10)
• Nilpotent operators and matrices ($T$ is nilpotent if means $T^m = 0$ for some $m \geq 1$)
• Generalized eigenspaces
• Characteristic polynomial of an upper-triangular matrix or of a linear transformation that admits one, i.e., if $V = \bigoplus \lambda V(\lambda)$, then $\chi_T(x) = \prod (x - \lambda)^{\dim V(\lambda)}$;
• Minimal polynomial (PS 10, # 8)

7) Computations involving eigenvalues and eigenspaces:
• Compute the eigenvalues of a two-by-two matrix using characteristic polynomial (and trace and determinant)
• Compute the eigenvalues of a larger square matrix $A$ using Gaussian elimination on $A - xI$.
• Given an eigenvalue, compute the eigenspace using Gaussian elimination.
• For a general linear transformation, find a basis in which its matrix is (block) upper triangular and compute this matrix.

8) Computations involving orthogonalization and orthogonal projections:
• Compute the orthogonal complement of a vector or a vector subspace
• Compute the orthogonal projection of a vector to a subspace
• Implement the Gram-Schmidt procedure to obtain an orthonormal basis from an arbitrary one by an upper-triangular change of basis.

9) Computation of polar and singular value decompositions
• By considering a matrix as a linear transformation, compute its singular value decomposition.
• Use the preceding to write a matrix in the form $U_1DU_2^{-1}$ where $D$ is diagonal with nonnegative entries, and $U_1$ and $U_2$ are unitary matrices.
• Use the SVD to compute the polar decomposition ($S = U_1U_2^{-1}$ and $P = U_2D_U^{-1}$)

10) Computation of generalized eigenspaces, characteristic polynomials, minimal polynomials:
• Given the eigenvalues, compute the generalized eigenspaces (using Gaussian elimination)
• Compute the characteristic polynomial of a linear transformation or matrix (if the vector space has dimension two or the transformation can be brought to upper-triangular form, i.e., if $V = \bigoplus \lambda V(\lambda)$);

11) Major results (Know the statements, but don’t memorize them word for word—know what they say mathematically. Only if mentioned below, know how to prove or demonstrate implications). Especially important ones are in bold.
• Existence of eigenvalues in complex case: Theorem 5.10
• Block upper-triangular matrices for linear transformations: Theorem 5.13 and slides for lecture 13
• Cauchy-Schwarz inequality: Theorem 6.6; triangle inequality: Theorem 6.9
• Gram-Schmidt orthogonalization: 6.20
• Orthogonal complements: Theorem 6.29
• Existence of adjoints: Theorem 6.45 and following discussion; see also Lecture 17
• The spectral theorem: Theorems 7.9, 7.13, and 7.25; see also lectures
• Polar decomposition: Theorem 7.41
• Singular Value Decomposition: Theorem 7.46
• The decomposition theorem: slides for Lecture 21; Theorem 8.23
• The Cayley-Hamilton theorem: Theorem 8.20, slides for Lecture 21
• Statement that $\dim(V(\lambda)) = d_\lambda$: Theorem 8.10; slides for Lecture 21
• Minimal polynomials: PS 10 # 8
• Square Roots: PS 10 # 9