On The Human Eye

Ramis Movassagh
Collegium Helveticum, Zurich
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I. Recap of Fourier Theory

Monsieur J. Fourier
His transformation relation

\[ \int_{-\infty}^{\infty} \exp(-i\omega t) f(t) dt = S(\omega) \]
Time Domain

\[ \int_{-\infty}^{\infty} \exp(-ik \cdot r) f(k) dk = S(r) \]
Momentum Space
Has to do with exploring nature

We learn about the “b box” by periodically perturbing it:
Eigenfunction of Linear time-invariant systems

\[ f_{\text{black box}} \left\{ \exp \left[ i \left( k \cdot r - \omega t \right) \right] \right\} = S(\omega) \cdot \exp \left[ i \left( k \cdot r - \omega t \right) \right] \]

\[ S(\omega) = \text{Eigenvalues} = \text{Spectrum (w)} = \text{Frequency Response Function} \]
Two views of same object

Conjugate Variables

FT
Quoted by Prof. Richard Ernst

“The most fruitful developments have happened whenever two different kinds of thinking were meeting”

- Werner Heisenberg
What else...

Generalize the Eigenfunctions

\[ \exp \left\{ i (k \cdot r - wt) \right\} \]

The world of Spectroscopy is quite rich. Integrating over:

Time (t): Time domain Spectroscopy

Space (r): Interferometry

Spatial frequency (k): Diffraction imaging
II. The Eye: quick overview

- Provides 80% of the information.

- 97% of the light directed at the pupil does not result in perception. Scattering and reflection.

For tons of info: See C. Oyster
The eye: refracting components

► RETINA: first encounter of the brain with light.
► If there is NOT an EYE → Point Spread Function on a film: Like Holographic domain. Like SNAILS, or OCELLI!
► EVOLUTION’S SOLUTION
► The lens and cornea: Emulate and propagate the sense of touch to distances.

Descartes’ depiction of the Eye’s functioning
Fourier relation for images

\[ F\{g\} = \iiint g(x, y) \exp \left[ -j2\pi \left( f_X x + f_Y y \right) \right] dx\, dy \]

\[ F^{-1}\{G\} = \iiint G(f_X, f_Y) \exp \left[ j2\pi \left( f_X x + f_Y y \right) \right] df_X\, df_Y \]

**CODE**

Fourier Transformation of Images

Way of compressing data (like JPEG)
III. Illusions

The image  F. T.  Phases
Importance of phases

Dock and the Fourier Dock

Cat and the Fourier Cat

Reconstruction from phase of the cat and the amplitude of dock
Could it be that the complexity of the Fourier Space is THE reason for the instability (-&gt; Illusions) of the visual system?
IV. Basic theoretical tools

► Geometrical Optics

[Diagram of Geometrical Optics with labels: Object, F1, F2, Focal Plane, Image Plane, Image]
Retina Image Quality

- Descartes’ classical model of the human eye.

*Guided by past experiments and common sense:*

- We believe in PLANE OF MAXIMUM INFORMATION behind the lens.

- Minimizing THE SHANNON ENTROPY $\Rightarrow$ maximize information.

Descartes’ model
V. Theoretical Approach

Consider the following schematics:

Integral that determines the image quality behind the lens.

\[
U(u, v, z) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(u^2+v^2)} \times \iint_{s^2} U_l(x, y) P(x, y) e^{\frac{jk}{2}(x^2+y^2)(\frac{1}{z}-\frac{1}{f})} e^{-j\frac{2\pi}{\lambda z}(xu+vy)} \, dx \, dy
\]

\[
U_l(x, y) = F^{-1} \left\{ F_0(f_x, f_y) e^{-j\pi d\lambda (f_x^2+f_y^2)} \right\}
\]

*P(x,y):* Pupil function

*U_l(x,y):* Input to the lens. That is the object after free propagation distance d.

F and F^{-1} refer to Fourier and Inverse Fourier transformations.

*F_0(f_x,f_y):* Fourier transformation of the input ‘object’
Minimizing Shannon’s Entropy

**Definition:** The concept of entropy in information theory describes how much randomness (or, alternatively, 'uncertainty') there is in a signal.

\[ H(x) = - \sum_{i=1}^{n} p(i) \log_2 p(i). \]

- **x:** an event
- **i:** all possible outcomes of x

Information and entropy relation:

\[ \therefore H \propto \frac{1}{I} \]

Claude Elwood Shannon (1916-2001)
Wavelets

- The world between the Fourier and Real space.
- One has to respect the geometry of the problem.
- For example the finiteness or the info content of the signal.
Typical Wavelet Basis

Mother Wavelets

2D dimensional Wavelets
Can we predict Center Surround Information Theoretically??!

Difference of Gaussian representing the CENTER-SURROUND (e.g. for Ganglion cells).
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