Optical "Bernoulli" Forces

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Curving of spinning balls in fluids

Curving soccer ball

J. W. M. Bush, “The aerodynamics of the beautiful game”
[http://math.mit.edu/~bush/?p=492]
Newton’s third law

When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction to that of the first body.
Rotation in a photonic fluid

\[ \Omega \]

\[ E \]

\[ z \]

\[ a \]

\[ x \]

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Michelson Morley: Constancy of speed of light
Michelson Morley: 1902 experiment
Albert Einstein: 1905
ON THE ELECTRODYNAMICS OF MOVING BODIES

By A. EINSTEIN

It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the motion of the magnet, when...
Rotation in a photonic fluid
Rotation in a photonic fluid

But: \( v_A^r = v_B^r \)
Rotation in a photonic fluid

\[ \Omega \]

\[ E \]

\[ z \]

\[ a \]

\[ x \]

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Suppose the fluid is a stream of photons (i.e., light):

- the *relative* speed is \( c \) regardless.
- massless fluid; no inertia
- very invicid
- carries energy and momentum
- couples to the electrical properties of matter
Previous work
Electrodynamics of moving media

Minkowski (1906-1907): Electrodynamics of *uniformly* moving media

\[
D = \varepsilon E \quad \rightarrow \quad D + v \times H/c = \varepsilon (E + v \times B/c)
\]

\[
B = \mu H \quad \rightarrow \quad B + E \times v/c = \mu (H + D \times v/c),
\]
Sommerfeld (see Electrodynamics 1952) argued that for \textit{rotating} axial symmetric bodies, same equations suffice ignoring $O\left(\frac{v}{c}\right)^2$. 
Sommerfeld (see Electrodynamics 1952) argued that for rotating axial symmetric bodies, same equations suffice ignoring $O\left(\frac{v}{c}\right)^2$.

We are solving a scalar Helmholtz equation in the three regions

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0$$
$E_{\text{tot}} = E_z \hat{z} = (E_i + E_s) \hat{z}$
Field Equations

\[ E_i = E_0 \exp(i k_0 r \cos \phi) \]
Field Equations

\[
E_i = E_0 \sum_{n=-\infty}^{+\infty} i^n J_n(k_0 r) \exp(i n \phi)
\]

\[
E_s = E_0 \sum_{n=-\infty}^{+\infty} i^n \alpha_n H_n^1(k_0 r) \exp(i n \phi)
\]

\[
E_t = E_0 \sum_{n=-\infty}^{+\infty} i^n \beta_n J_n(\gamma_n r) \exp(i n \phi)
\]

C.T. Tai (1964)
Eliminating $\beta_n$:

$$\alpha_n = -\frac{J_n(\rho_0)[J_{n-1}(\rho_n) - J_{n+1}(\rho_n)] - k_0 \gamma_n J_n(\rho_n)[J_{n-1}(\rho_0) - J_{n+1}(\rho_0)]}{H_n^{(1)}(\rho_0)[J_{n-1}(\rho_n) - J_{n+1}(\rho_n)] - k_0 \gamma_n J_n(\rho_n)[H_{n-1}^{(1)}(\rho_0) - H_{n+1}^{(1)}(\rho_0)]}$$

$$\gamma_n^2 = k^2 - 2n\omega K = k^2 \left(1 - \frac{2nm\Omega}{\omega}\right)$$

$$m = 1 - \frac{\varepsilon_0}{\varepsilon}$$

$$K = \mu_0 (\varepsilon - \varepsilon_0)\Omega$$
$E_{tot} = E_z \hat{z}$
Maxwell Stress Tensor in Free Space

\[ \mathbf{\sigma} = \varepsilon_0 \mathbf{E} \otimes \mathbf{E} + \mu_0 \mathbf{H} \otimes \mathbf{H} \]

\[ - \frac{1}{2} \left( \varepsilon_0 \mathbf{E}^2 + \mu_0 \mathbf{H}^2 \right) \left( \mathbf{\hat{x}} \otimes \mathbf{\hat{x}} + \mathbf{\hat{y}} \otimes \mathbf{\hat{y}} + \mathbf{\hat{z}} \otimes \mathbf{\hat{z}} \right) \]
To calculate the force on the cylinder in any direction $\hat{n}_0$ on the plane at a fixed radius $r_0$, we evaluate

$$F_{\hat{n}_0} = \frac{\omega r_0}{2\pi} \int_{0}^{\frac{2\pi}{\omega}} dt \int d\phi \left\{ \hat{n}_0 \cdot \overrightarrow{\sigma} \cdot \hat{r} \right\},$$

(2)
Normalized force

To quantify the strength of force we normalize by the incident power on the scatterer’s geometrical cross section

\[ F_{\hat{n}_0} \rightarrow \frac{cF_{\hat{n}_0}}{P} \]

\[ P = 2a|E_0|^2 \sqrt{\frac{\varepsilon_0}{\mu_0}} \]
The Problem Statement

Previous work

Our results

Applications and experimental feasibility

Normalized force

\[ \frac{cF}{P} = 2 \]

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Normalized force on a dielectric: $\varepsilon/\varepsilon_0 = 10$ and $a = \lambda = 1$ cm
How about metals?

Eliminating $\beta_n$:

$$\alpha_n = -\frac{J_n(\rho_0)[J_{n-1}(\rho_n)-J_{n+1}(\rho_n)] - k_0 \gamma_n J_n(\rho_n)[J_{n-1}(\rho_0)-J_{n+1}(\rho_0)]}{H_n^{(1)}(\rho_0)[J_{n-1}(\rho_n)-J_{n+1}(\rho_n)] - k_0 \gamma_n J_n(\rho_n)[H_{n-1}^{(1)}(\rho_0)-H_{n+1}^{(1)}(\rho_0)]}$$

$$\gamma_n^2 \equiv k^2 - 2n\omega K = k^2 \left(1 - \frac{2nm\Omega}{\omega}\right)$$

$$m \equiv 1 - \frac{\varepsilon_0}{\varepsilon}$$

$$K \equiv \mu_0 (\varepsilon - \varepsilon_0) \Omega$$
Normalized force: negative $\chi$, e.g., metals
Mie resonances

\[ cF_y / P \]

\[ a / \lambda_0 \]

\[ 10^{-2} \quad 10^{-1} \quad 10^{0} \quad 10^{1} \quad 10^{2} \]

\[ -0.025 \quad -0.02 \quad -0.015 \quad -0.01 \quad -0.005 \quad 0 \quad 0.005 \quad 0.01 \quad 0.015 \quad 0.02 \]

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• Combined with optical tweezers


it is possible to produce a *purely lateral* force.
• Combined with optical tweezers

  it is possible to produce a purely lateral force.

• *Superscattering*: Can be resonantly enhanced by using multilayer spheres: align multiple resonances at the same frequency.

  Z. Ruan and S. Fan, Appl. Phys. Lett. 98, 043101 (2011).
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• Material dispersion: Similar to the origin of quantum friction. The Doppler shift in the material dispersion should differ top and bottom
  R. Zhao, A. Manjavacas, F. J. García de Abajo, and J. B. Pendry, Phys. Rev. Lett. 109, 123604 (2012)
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• Near GHz Ω rotation of graphene particles

Future Work

- Full specification of amplitude and phase of the light fields
- Suggesting indirect experiments to probe it using interferometry
Future Work: Phases
Optical Bernoulli Forces Could Steer Objects Bathed in Light, Say Theorists

Theorists have discovered a new optical force that is analogous to the thrust that keeps aircraft aloft and causes tennis balls to swerve.

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