18.703 HOMEWORK #6, DUE TUESDAY APRIL 3RD

1. Find a presentation of the dihedral group $D_n$.  
2. Herstein, Chapter 2, §9, 2.  
3. Herstein, Chapter 3, §3, 1.  
4. Herstein, Chapter 3, §3, 2.  
5. Herstein, Chapter 3, §3, 3.  
6. Herstein, Chapter 3, §3, 6: Show that the 3-cycles generate $A_n$.  
8. Let $P$ be a Sylow $p$-subgroup of a finite group $G$. Show that $P$ is characteristically normal in $G$ if and only if there is only one Sylow $p$-subgroup.  
9. Let $G$ be an abelian group of order $p^l q^m$, where $p$ and $q$ are distinct primes and $l > 0$, $m > 0$. Show that $G \simeq P \times Q$, where $P$ is a subgroup of order $p^l$ and where $Q$ is a subgroup of order $q^m$.  
10. Let $G$ be a simple group of order 168. Show that $G$ is isomorphic to a subgroup of $S_8$.  
11. Show that every simple group whose order lies between 1 and 59 has prime order.  
12. Challenge Problem: Herstein, Chapter 3, §3, 9 & 10:  
   (i) If $n \geq 5$ and $\{ e \} \neq N < A_n$ is a normal subgroup, show that $N$ must contain a 3-cycle.  
   (ii) Show that $A_n$ is simple if and only if $n \neq 4$.  
13. Challenge Problem: Show that every simple group whose order lies between 61 and 167 has prime order.