FOURTH PRACTICE MIDTERM A  
MATH 18.02, MIT, AUTUMN 12

You have 50 minutes. This test is closed book, closed notes, no calculators.

There are 5 problems, and the total number of points is 100. Show all your work. Please make your work as clear and easy to follow as possible.

Name:______________________________

Signature:__________________________

Student ID #:_______________________

Recitation instructor:________________

Recitation Number+Time:______________

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1. (20pts) Let $D$ be the domain in the first octant cut off by the plane $3x + 2y + z = 1$. Assume the density $\delta = z$.
(a) Set up an iterated integral in rectangular coordinates for the total mass of $D$.

(b) Evaluate only the inner integral
2. (20pts) A solid hemisphere has radius $a$ and density 1 and it is placed so its flat side is on the $xy$-plane, with the centre at $(0, 0)$. Set up and evaluate a triple integral in spherical coordinates which gives its gravitational attraction on a unit point mass at the origin $(0, 0, 0)$. 
3. (20pts) Consider the surface $S$ given by the equation
\[ z = (x^2 + y^2 + z^2)^2. \]
(a) Show that $S$ lies in the upper half space $z \geq 0$.

(b) Write out the equation for this surface in spherical coordinates.

(c) Write down an iterated integral for the volume of the region inside $S$. 
4. (20pts) Let $S$ be the portion of the ellipsoid $x^2 + y^2 + 3z^2 = 1$ that lies above the $xy$-plane. Let

$$\vec{F} = (x + y^3)i + (2y - e^x)j - (3z + 1)k.$$ 

Compute the flux of $\vec{F}$ through $S$ (orient $S$ upwards).
5. (20pts) Let $S$ be the part of the surface $z = xy$ where $x^2 + y^2 < 1$. Compute the flux of

$$\vec{F} = y\hat{i} + x\hat{j} + z\hat{k},$$

upward across $S$. 