THIRD PRACTICE MIDTERM A
MATH 18.02, MIT, AUTUMN 12

You have 50 minutes. This test is closed book, closed notes, no calculators.

There are 6 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.*

Name:__________________________
Signature:_______________________
Student ID #:____________________
Recitation instructor:_____________
Recitation Number+Time:__________

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1
1. (15pts) Evaluate
\[ \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \frac{1}{1 - y} \, dy \, dx. \]
2. (10pts) Find the $x$-coordinate $\bar{x}$ of the centre of mass of the portion of the unit disk in the first quadrant, bounded by the $x$-axis and the line $y = x$ (assume the density $\delta = 1$).
3. (20pts) (i) Find $a$ such that
\[ \vec{F} = (x^2 - 14xy)\hat{i} + (ax^2 - 2y)\hat{j}. \]
is conservative.

(ii) Find a potential function $f(x, y)$ for $\vec{F}$ for this value of $a$.

(iii) Calculate the line integral
\[ \int \vec{F} \cdot d\vec{r}, \]
for the curve $C$ given by $x = 3\sin t$, $y = \cos t$, $0 \leq t \leq \pi$, where $C$ has the orientation which starts at $(0, 1)$. 
4. (20pts) Consider the region in the $xy$-plane bounded by the curves 
$y = x^2$, $y = x^2/3$, $xy = 3$ and $xy = 5$.
(i) Compute $du \, dv$ in terms of $dx \, dy$, where $u = x^2/y$ and $v = xy$.

(ii) Find a double integral for the area of $R$ in $uv$-coordinates and evaluate it.
5. (20pts) Let $C$ be the positively oriented closed curve formed by the parabola $y = x^2$ running from $(-1, 1)$ to $(1, 1)$ and by a horizontal line segment running from $(1, 1)$ back to $(-1, 1)$. Evaluate

$$\oint_C y^2 \, dx + 4xy \, dy,$$

(i) directly, and

(ii) by using Green’s theorem.
6. (15pts) Find the volume of the region enclosed by the plane $z = 4$ and the surface
\[ z = (2x - y)^2 + (x + y - 1)^2. \]