18.02 HOMEWORK #9, DUE THURSDAY NOVEMBER 8TH

Part A (15 points)

(11/01) Notes V2; 15.3
4C/5ab, 6ab
(11/02) 15.4 to top of page 1043.
4D/1abc, 2, 3, 4, 5.
(11/06) Notes V3, V4.
4E/1ac, 2, 3, 4, 5.
4F/2, 3, 4.

Part B (29 points)

1. (Thursday 3 points: 1+2) Continued from question 5, Homework # 8.
(iv) Show that the curl of $\vec{F}$ is zero at any point of the plane where $\vec{F}$ is defined (not just in the right half plane $x > 0$).
(v) Is $\vec{F}$ conservative over its entire domain of definition? Is it conservative over the right half plane $x > 0$? Justify your answer.

2. (Thursday, 4 points: 2+2) (i) Calculate the curl of $\vec{F} = r^n(x\hat{i} + y\hat{j})$ (where $r = \sqrt{x^2 + y^2}$; start by finding formulas for $r_x$ and $r_y$).
(ii) Whenever possible, find a potential $g$ such that $\vec{F} = \nabla g$ (Hint: look for a potential of the form $g = g(r)$. Pay attention to rogue values of $n$)

3. (Friday 4 points: 2+2) (i) Show that if a simple closed (positively oriented) curve $C$ is the boundary of a region $R$ then
$$\text{area}(R) = \oint_C x \, dy = \oint_C -y \, dx.$$ 
(ii) Find the area of the region between the $x$-axis and one arch of the cycloid with parametric equations
$$x = a(t - \sin t) \quad \text{and} \quad y = a(1 - \cos t).$$

4. (Friday 6 points: 4+2) (i) For what simple closed (positively oriented) curve $C$ in the plane does the line integral
$$\oint_C (x^2 y + y^3 - y) \, dx + (3x + 2y^2 x + e^y) \, dy$$
have the largest possible value?
(ii) What is the maximum value?

5. (Friday 6 points: 3+3) For each statement below, say whether it is TRUE or FALSE. If it is true, explain why; if false give an example to show that it is definitely false.
(i) If $\vec{F}$ and $\vec{G}$ are conservative vector fields, then $\vec{F} + \vec{G}$ is a conservative vector field.

(ii) If $M$ and $N$ are differentiable functions on the region $R$, given by $1 < x^2 + y^2 < 4$ and $M_y(1, -1) \neq N_x(1, -1)$, then $(M, N)$ is not a gradient vector field.

6. (Tuesday, 6 points: 2+2+2) (i) Let $C$ be the unit circle, oriented counterclockwise, and consider the vector field $\vec{F} = xy\hat{i} + y^2 \hat{j}$. Which portions of $C$ contribute positively to the flux of $\vec{F}$? Which portions contribute negatively?

(ii) Find the flux of $\vec{F}$ through $C$ by direct calculation (evaluating a line integral). Explain your answer to (i).

(iii) Find the flux of $\vec{F}$ through $C$ using Green’s theorem.

**Part C: 0 points**