PART A (17 POINTS)

(11/15) Notes I4, CV.4, G; 14.7 pages 990-992.
12.8/9, 11, 15, 27, 55 (note that there is a mistake in the book; secant should be cosecant.)
5B/1abc, 2, 3, 4abc
5C/2, 3, 4.
6A/1, 2, 3, 4
6B/1, 2, 3, 4, 6, 8.
(11/20) V10, 15.6
6C/1a, 2, 3, 5, 6, 7a, 8.
(11/27) V10, 15.6, pages 1054-1055.

PART B (23 POINTS)

1. (Thursday 4 points) Write down and evaluate integrals, in both cylindrical and spherical coordinates, for the average distance from a point of the solid sphere of radius \( a \) to a fixed point \( O \) on the surface of the sphere. (Hint: put the point \( O \) at the origin and position the sphere so the origin is the south pole of the sphere.)

2. (Thursday 4 points) 5C/5.

3. (Friday 2 points) 6B/7.

4. (Tuesday 8 points: 1+2+3+2) Consider a tetrahedron with vertices at \( P_0 = (0, 0, 0) \), \( P_1 = (1, 0, 1) \), \( P_2 = (1, 0, -1) \) and \( P_3 = (1, 1, 0) \).
   (i) Which two faces are exchanged by the symmetry \( z \rightarrow -z \)?
   (ii) Find normals to each face (no need to write down unit normals) pointing outwards.
   (iii) Calculate the flux of \( \vec{F} = -y\hat{j} \) through each face.
   (iv) Check the divergence theorem for this vector field and this solid, by computing each side of the formula.

5. (Tuesday 5 points: 2+1+2) Let
   \[ f(x, y, z) = \frac{1}{\rho} = (x^2 + y^2 + z^2)^{-1/2}. \]
   (i) Calculate \( \vec{F} = \nabla f \) and describe geometrically the vector field \( \vec{F} \).
   (ii) Evaluate the flux of \( \vec{F} \) over the sphere of radius \( a \) centred at the origin.
   (iii) Show that \( \text{div} \vec{F} = 0 \). Does the answer to (ii) contradict the divergence theorem? Explain.
Use the $(3 \times 3)$ Jacobian to give another way to see that
\[ \Delta V \approx \rho^2 \sin \phi \Delta \theta \Delta \phi \Delta \rho. \]