(1) (6.1.1)
(a) \(x' = (-3, 4)\) so \(||x'|| = 5\), hence
\[
\int_x f ds = 5 \int_0^2 (2 - 3t + 8t - 2) dt = 50.
\]
(b) \(x' = (-\sin t, \cos t)\) so \(||x'|| = 1\), hence
\[
\int_x f ds = \int_0^{\pi} (\cos t + 2 \sin t) = 4.
\]

(2) (6.1.3) \(x' = (1, 1, 3\sqrt{t}/2)\) so \(||x'|| = \sqrt{2 + 9t/4}\), hence
\[
\int_x f ds = \int_1^{35/4} \frac{t + t^{3/2}}{t + t^{3/2}} \sqrt{2 + 9t/4} dt.
\]

We perform the substitution \(t = \frac{4}{9}(u - 2)\) and get that this equals
\[
\int_{17/4}^{35/4} 4\sqrt{u}/9 du = \frac{1}{27} [17^{3/2} - 35^{3/2}] .
\]

(3) (6.1.7) \(x' = (\cos t, \sin t)\), hence
\[
\int_F F \cdot ds = \int_0^{\pi/2} \left[(-\cos t + 2) \cos t + \sin^2 t \right] dt.
\]

It is easy to see by change of variable that \(\int_0^{\pi/2} \sin^2 t = \int_0^{\pi/2} \cos^2 t\) and so the above integral equals
\[
\int_0^{\pi/2} 2 \cos t dt = 2.
\]

(4) (6.1.11) \(x' = (-3 \sin 3t, 3 \cos 3t)\), hence
\[
\int_x x dy - y dx = \int_0^{\pi} 3 \cos^2 3t + 3 \sin^2 3t = 3\pi.
\]

(5) (6.1.13) \(x' = (2e^{2t} \cos 3t - 3e^{2t} \sin 3t, 2e^{2t} \sin 3t + 3e^{2t} \cos 3t)\), hence
\[
\int_x x dx + y dy = \int_0^{2\pi} \frac{2e^{2t}}{e^{6t}} = 1 - e^{-4\pi}.
\]

(6) (6.1.16) A parametrization of the curve (there is more than one) is \(x(t) = (t, 5 - 4t, 2t - 1)\) for \(1 \leq t \leq 2\). We have \(x' = (1, -4, 2)\), hence the work is
\[
\int_1^2 [t^2(5 - 4t) - 4(2t - 1) + 2(6t - 5)] dt = \int_1^2 [-4t^3 + 5t^2 + 4t - 6] dt = -\frac{31}{3} .
\]
(7) (6.1.19) Parameterize $C$ by a curve $x$ defined by

$$x(t) = \begin{cases} 
(t, t) & 0 \leq t \leq 1, \\
(t, 1) & 1 < t \leq 3, \\
(3, 4 - t) & 3 \leq t \leq 4, \\
(7 - t, 0) & 4 \leq t \leq 7.
\end{cases}$$

Note that this curve have clockwise orientation, so we will remember to take $-1$ to whatever we get in the integral. We have

$$dx = \begin{cases} 
1 & 0 \leq t \leq 3, \\
0 & 3 \leq t \leq 4, \\
-1 & 4 \leq t \leq 7.
\end{cases}$$

and

$$dy = \begin{cases} 
1 & 0 \leq t \leq 1, \\
0 & 1 \leq t \leq 3, \\
-1 & 3 \leq t \leq 4, \\
0 & 4 \leq t \leq 7.
\end{cases}$$

Thus

$$\int_C x^2y \, dx - (x + y) \, dy = -\int_0^1 t^3 \, dt - \int_1^3 t^2 \, dt + \int_0^1 2t \, dt - \int_3^4 (7 - t) \, dt = -\frac{137}{12}.$$

(8) (6.1.21) A parametrization of the curve is $x(t) = (1 + 4t, 1 + 2t, 2 - t)$ for $0 \leq t \leq 1$, so $x' = (4, 2, -1)$. Hence

$$\int_C yz \, dx - xy \, dy + xy \, dz = \int_0^1 [4(1 + 2t)(2 - t) - 2(1 + 4t)(2 - t) - (1 + 4t)(1 + 2t)] \, dt = -\frac{11}{3}.$$

(9) (6.2.8) Let $D$ be the ellipse. We have $N_x = 1, M_y = -4$, so by Green’s theorem the work equals

$$\int \int_D -3 \, dy \, dx = -12 \pi.$$

(10) (6.2.10) Let $F(x, y) = (0, x)$ so that $N_x - M_y = 1$. Then by Green’s theorem, the area is $-\int_0^{2\pi} a^2(t - \sin t) \, dt = 3\pi a^2$. (Since $F$ is 0 on the x-axis)

(11) (6.2.11) $C$ is negatively oriented, and $N_x - M_y = 5$. So by Green’s theorem, the integral is just $-5 \times \text{Area} = -45$.

(12) (6.2.14) We need to subtract the area of the ellipse from $25 \pi$. Take $F(x, y) = (0, x)$ so that $N_x - M_y = 1$. Then by Green’s theorem, the area of the ellipse is the line integral of $F$ on the boundary of the ellipse. Let $(x, y) = (3 \cos t, 2 \sin t)$. The area of the ellipse is $\int_0^{2\pi} 6 \cos^2 t \, dt = 6 \pi$. Hence the area between the circle and the ellipse is $19 \pi$.

(13) (6.2.19) The integrand vector field is smooth everywhere. Since $N_x - M_y = 3x^2 - 3x^2 = 0$, by Green’s theorem, the integral is 0.

(14) (6.2.20) The integrand vector field is smooth everywhere. Since $N_x - M_y = 3x^2 + 2 + 3y^2 > 0$ for all $x, y$, by Green’s theorem, the integral has the same value as the double integral of a positive function. Hence it’s always positive.
(15) (6.2.25) Let $F = \nabla f$. Then by the divergence theorem in the plane, we have
\[ \oint_{\partial D} \nabla f \cdot \mathbf{n} ds = \int \int_{D} \nabla^2 f dA \]