
2. Show that any second-order linear constant coefficient differential operator on $C^2(\mathbb{R}^n)$ of the form

$$L = \sum_{i,j=1}^{n} a_{ij} \frac{\partial^2}{\partial x^i \partial x^j} + \text{lower order}$$

can be expressed, in appropriate coordinates $y = (y^1, \ldots, y^n)$, as

$$L = \sum_{i,j=1}^{n} \hat{a}_{ij} \frac{\partial^2}{\partial y^i \partial y^j} + \text{lower order}$$

where the matrix $\hat{a}_{ij}$ is diagonal and $\hat{a}_{ii} \in \{1, 0, -1\}$ for $i = 1, 2, \ldots, n$.

3. Prove that Laplace’s equation, $\Delta u = 0$, is rotationally invariant, that is, if $u(x)$ is a solution, then so is $v(x) = u(Qx)$ for any orthogonal $n \times n$ matrix $Q$.


6. Suppose $u(x, t)$ solves

$$\begin{cases}
    u_{tt} - c^2 u_{xx} = 0 & (x, t) \in \mathbb{R} \times (0, \infty), \\
    u(x, 0) = \phi(x) & x \in \mathbb{R}, \\
    u_t(x, 0) = \psi(x) & x \in \mathbb{R}.
\end{cases}$$

(a) Suppose $\psi \equiv 0$ and supp $\phi \subset [-2, 2]$. Let $f(t) = u(100, t)$. If $\phi(x) = \cos 2\pi x$ on $[-1, 1]$, at what time(s) can you determine the value of $f(t)$? (b) For what, if any, positive $t$ is $f(t)$ necessarily 0?

(c) If instead $\phi \equiv 0$ and supp $\psi \subset [-2, 2]$, with $\psi(x) = \sin 2\pi x$ on $[-1, 1]$, does the set of $t$ for which you can determine $f(t)$ change?