18.152 – Problem Set 1

1. Strauss 1.1, pp. 5, 6. # 7, 10.

2. Strauss 1.2, pp. 9, 10. # 7, 13.

3. Strauss 1.3 pp. 19, 20. # 6, 8

4. Suppose $E(x, t)$ and $B(x, t)$ satisfy Maxwell’s equations:

$$
\begin{align*}
E_t &= \text{curl } B, \\
B_t &= -\text{curl } E, \\
\text{div } B &= \text{div } E = 0.
\end{align*}
$$

Show that each of the components $B^i, E^i, i = 1, 2, 3$ of $B$ and $E$ satisfy the wave equation $u_{tt} - \Delta u = 0$.

5. (a) Use the divergence theorem to prove that if $W \in C^1(\mathbb{R}^n, \mathbb{R}^n)$ satisfies

$$
|W(x)| \leq \frac{C}{|x|^n + 1}
$$

then

$$
\int_{\mathbb{R}^n} \text{div } W \, dV = 0.
$$

(See Strauss 1.3, # 10)

(b) Use (a) to prove the following important integration-by-parts identity for $u \in C^1(\mathbb{R}^n), v \in C^1_c(\mathbb{R}^n)$:

$$
\int_{\mathbb{R}^n} \frac{\partial u}{\partial x^i} v \, dV = -\int_{\mathbb{R}^n} u \frac{\partial v}{\partial x^i} \, dV,
$$

$i = 1, 2, \ldots, n.$