1. **4.1.1** (a), (b), (d), **4.1.2**.

2. (a) Let $f(x)$ be a periodic function of period $2\pi$ with continuous first and second order derivatives, and $\overline{f} = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)dx$. Show that 
\[
\int_{-\pi}^{\pi} (f'(x))^2 \, dx \geq \int_{-\pi}^{\pi} (f(x) - \overline{f})^2 \, dx,
\]
and, if 
\[
\int_{-\pi}^{\pi} (f'(x))^2 \, dx = \int_{-\pi}^{\pi} (f(x) - \overline{f})^2 \, dx,
\]
then $f(x)$ is of the form $f(x) = a + b \cos x + c \sin x$, where $a$, $b$ and $c$ are constants.

(b) Let $r$ be a simple closed curve on $\mathbb{R}^2$ of length $2\pi$, and $r(s) = (x(s), y(s))$, $-\pi \leq s \leq \pi$, the arc length parametrization of $r$. Assume the area inside $r$ is $A$. Show that 
\[
A = \int_{-\pi}^{\pi} (x(s) - \overline{x})y'(s)ds,
\]
where $\overline{x} = \frac{1}{2\pi} \int_{-\pi}^{\pi} x(s)ds$.

(c) Use (a) and (b) to prove that $A \leq \pi$, and, if $A = \pi$, then $r$ is a circle of radius 1. (Hint: use the facts that $2\pi = \int_{-\pi}^{\pi} ds$ and $(\frac{dx}{ds})^2 + (\frac{dy}{ds})^2 = 1$.)