HOMEWORK 1

1. 3.1.1, 3.1.7.

2. Show that any second order linear differential equation $u'' + R(x)u' + S(x)u = T(x)$ is equivalent to an equation of the standard form $-\frac{d}{dx}(c(x)\frac{du}{dx}) + q(x)u = f(x)$.

3. Consider the boundary value problem

$$\begin{cases} -u'' + \lambda u = 0 \\ u(0) = u(1) = 0 \end{cases} \quad (1),$$

where $\lambda$ is a real constant.

(a) Show that, if (1) has non zero solutions, then $\lambda$ is negative. (Hint: Use the inner product $(u, v) = \int_0^1 u(x)v(x)dx$.)

(b) Find all the real numbers $\lambda$, for which (1) has non zero solutions. (Hint: Write down the formula of the general solution, and check the boundary conditions.)