Problem Set 2

Due: Wednesday October 19

Prove 5 of 6 of the following.

1. Every $n$-vertex graph containing no path with 3 edges has at most $n$ edges.

2. Every $n$-vertex graph with minimum degree at least $n/2$ contains a cycle of length $n$.

3. Every 3-uniform hypergraph $G$ on $n$ vertices such that no 6 vertices contains 3 edges has $o(n^2)$ edges.

4. Every graph on $n \geq 2$ vertices contains an $\epsilon$-regular pair with each part of order at least $\delta n$ with $\delta = 2^{-\epsilon - O(1)}$.

5*. For every graph $H$ and $\epsilon > 0$, there is $\delta = \delta(\epsilon, H) > 0$ such that every graph on $n$ vertices without an induced copy of $H$ contains an induced subgraph on at least $\delta n$ vertices which has edge density at most $\epsilon$ or at least $1 - \epsilon$.

6*. For each $c > 0$ there is $c' > 0$ such that every graph $G$ with $n$ vertices and $cn^2$ edges contains a $d$-regular subgraph with $d \geq c'n$. 