1. Consider the following sets on the real line \( \mathbb{R} \).

1) \( \{ \frac{m}{n} | m, n \in \mathbb{Z}, m > 0, n > 2^m \} \).
2) \( \{ \frac{m}{n} | m, n \in \mathbb{Z}, m > 0, n > 2^m \} \cup \{0\} \).
3) \( \{ \frac{m}{n} | m, n \in \mathbb{Z}, m > 0, n > 2^m \} \cup \{1\} \).
4) \( \{ n!/2^m | n \geq 1 \} \).
5) \( [\sqrt{2}, \sqrt{3}] \cap \mathbb{Q} \).

6) The set of real numbers with decimal expansion \( 0.x_1x_2... \) where \( x_i = 3 \) or \( 5 \).
7) The intersection of set (6) with \( \mathbb{Q} \).
8) The set of rational numbers which can be written with odd denominator.
9) The set of real numbers \( r \) such that there exists a rational number \( q = m/n \) (\( n > 0 \)) such that \( |r - q| < 1/10^n \).

Write down \(^1\) (without proof) the numbers of all of the above sets which are

(a) closed in \( \mathbb{R} \):
(b) open in \( \mathbb{R} \):
(c) compact:
(d) perfect (as a subset of \( \mathbb{R} \)):
(e) dense in \( \mathbb{R} \):
(f) closed in \( \mathbb{Q} \):
(g) open in \( \mathbb{Q} \):
(h) bounded:
(i) countable:

\(^1\)Each question is worth 3 points. Each mistake or omission is \(-2\) points. If you have a negative total, you get 0.
2. Let $E$ be a closed subset of $\mathbb{R}$.
   (a) (5 points) Show that the set of isolated points of $E$ is at most countable.
   (b) (10 points) Construct a set $F$ containing $E$ whose set $F'$ of limit points is $E$. 
3. (15 points) Let \( \{d_n, n \geq 1\} \), be a sequence of positive numbers. Let \( X \) be the set of nonnegative integers \( \mathbb{Z}_{\geq 0} \) with metric defined by \( d(0, m) = d_m \) for \( m \neq 0 \) and \( d(m, n) = d_m + d_n \) if \( m, n \neq 0 \) and \( m \neq n \). For which sequences \( \{d_n\} \) is \( X \) compact?

Hint. \( d_n = 1 \) and \( d_n = 1/n \) are useful examples to consider.