18.100B Problem Set 5

Due in class Monday, March 16. You may discuss the problems with other students, but you should write solutions entirely on your own.

1. Text, page 78, number 7. (Hint: the Schwartz inequality Theorem 1.35 can help.)

2. Text, page 81, number 16.

3. Suppose that $X$ and $Y$ are metric spaces, $f: X \to Y$ is any function, and $p \in X$. Show (using Definition 4.5) that $f$ is continuous at $p$ if and only if the following condition is satisfied: for every sequence $(p_n)$ in $X$ that converges to $p$, we have

$$\lim_{n \to \infty} f(p_n) = f(p)$$

in $Y$.

4. (If you’ve seen one convergent sequence, you’ve seen them all.) Let $X$ be the metric space consisting of the points $\{1/n| n = 1, 2, 3, \ldots\}$ and 0 in $\mathbb{R}$, with the distance function coming from $\mathbb{R}$. Let $Y$ be any metric space, $(p_n)$ a sequence in $Y$, and $p_0$ any point of $Y$. Define a function $f: X \to Y$ by

$$f(1/n) = p_n, \quad f(0) = p_0.$$ 

Show that $f$ is continuous if and only if the sequence $(p_n)$ converges to $p_0$. 