Problem Set 10

Due December 6th at 4 pm in room 2-285.

Hand in parts 1 and 2 separately. Put your name on each part.

Part 1

1. Let \( K: [0, 1] \times [0, 1] \to \mathbb{R} \) be continuous, and let \( \mathcal{F} \) be the family of functions \( f \) from \([0, 1]\) to \( \mathbb{R} \) satisfying

\[
    f(x) = \int_0^1 K(x, y)g(y)\,dy
\]

for some continuous function \( g: [0, 1] \to [-1, 1] \). Prove that the family \( \mathcal{F} \) is equicontinuous.

2. Problem 15 from page 168.


Part 2

4. Problem 1 from page 196.

5. Prove that if \( \alpha \in \mathbb{R} \) and \( f: (0, \infty) \to \mathbb{R} \) is given by \( f(x) = x^\alpha \), then \( f'(x) = \alpha x^{\alpha-1} \).

6. Problem 10 from page 139, parts (a) through (c). Hint: Use the results of the previous Problem and of Problem 7 from Problem Set 8.