Introduction to Proofs
IAP 2015
In-class problems for day 10

Problem 16. Use the Cauchy-Schwarz inequality to prove that for all $a, b, c \in \mathbb{R}$,
\[ a^2 + b^2 + c^2 \geq ab + bc + ca. \]

Proof.

Problem 17. Fix $n \in \mathbb{N} \setminus \{0\}$. Use the Cauchy-Schwarz inequality to prove that if $a_1, a_2, \cdots, a_n$ are a given collection of $n$ real numbers, then the inequality
\[ \sum_{i=1}^{n} \frac{a_i^2}{h_i} \geq \left( \frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} h_i} \right)^2 \]
holds for all $h_1, \cdots, h_n \in \mathbb{R}$ with each $h_i > 0$.

Proof.