Problem 1, Section 4.1, exercises 7 – 12

Problem 2, Exercises:

1. In section 4.1, exercise 8, let $A_t \in S_n$ be a family of linear mappings depending smoothly on $t$ and satisfying $A_t^2 = A_t$. Show that if $A_0 = A$ and $B = \frac{d}{dt} A_t(t = 0)$ then $BA + AB = B$.

2. Conclude from this that if $V$ is the kernel of $A$ and $W$ the kernel of $I - A$, then $B$ maps $V$ into $W$ and $W$ into $V$.

3. Let $\langle v, w \rangle$ be the Euclidean inner product on $\mathbb{R}^n$. Show that since $B$ is in $S_n$, $\langle Bv, w \rangle = \langle v, Bw \rangle$ for all $v \in V$ and $w \in W$, and conclude that $B$ is determined by its restitution to $V$.

4. Show that if $A$ is in $G(k, n)$, the tangent space to $G(k, n)$ at $A$ is $\text{Hom}(V, W)$.