# 18.706 HOMEWORK 5

DUE OCT.7, 2020

## Theorems

In this part you will see theorems that we studied in the class. Please supply a detailed proof of each.

**Theorem 1.** *Let $G$ be a group and $k$ be a field of characteristic zero. Let $V_1, V_2$ be two finite-dimensional representations of $G$ over $k$ with the same characters. Then the semisimplifications of $V_1$ and $V_2$ are isomorphic.*

Note: the semisimplification of $V$ is $\oplus(S^{\oplus m_S(V)})$, where the sum of is over all irreducible representations of $G$ over $k$, and $m_S(V)$ is the multiplicity of $S$ in any composition series of $V$.

## Exercises

**Problem 1.** Let $C_n$ be the cyclic quiver with $n$ vertices, i.e., $C_n$ has vertices $v_1, \cdots, v_n$ and arrows $e_i : v_i \to v_{i+1}$ for $1 \leq i \leq n$ (convention: $v_{n+1} = v_1$). Let $k$ be an algebraically closed field.

(1) Classify simple representations of $C_n$ over $k$.

(2) (Optional) Classify indecomposable representations of $C_n$ over $k$.

**Problem 2.** Let $k$ be a field of characteristic $p > 0$ and $A_n(k)$ be the Weyl algebra $k\langle x_1, \cdots, x_n, \partial_1, \cdots, \partial_n\rangle$ with relations $[\partial_i, x_j] = \delta_{ij}$, $[x_i, x_j] = 0$ and $[\partial_i, \partial_j] = 0$ for all $1 \leq i, j \leq n$.

(1) Show that $x_i^p$ and $\partial_i^p$ are in the center of $A_n(k)$.

(2) Show that $Z(A_n(k))$ is isomorphic to the polynomial ring over $k$ with free generators $\{x_i^p, \partial_i^p\}_{1 \leq i \leq n}$.