Problem 1.
Out of the students in a class, 60% like the Celtics, 70% like the Patriots, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a Celtics nor a Patriots fan.

Problem 2.
(a) How many possible strings of 5 letters can be formed with the letters A-Z?

(b) If you believe bestwordlist.com that there are 12,478 words in the English language with exactly 5 letters, what is the probability that a randomly selected 5-letter string is a valid English word?

Problem 3.
(a) In four rolls of a fair die, what is the probability that there will be at least one 6 rolled?

(b) In 24 rolls of a pair of fair dice, what is the probability that there will be at least one double-6 rolled?

(Historical note: this is known as de Méré’s Problem, and dates back to the mid-1600s; its solution is sometimes credited with laying the foundations of modern probability theory.)

Problem 4.
There are three random people in a room. What is the probability you can find two people who have their birthdays in the same month? (Assume that all months are equally long.)

Problem 5.
(a) Find the number of quadruples \((x_1, x_2, x_3, x_4)\) of positive integers such that

\[x_1 + x_2 + x_3 + x_4 = 20.\]

(b) Find the number of quadruples \((x_1, x_2, x_3, x_4)\) of non-negative integers such that

\[x_1 + x_2 + x_3 + x_4 = 16.\]

(c) Find the number of triples \((x_1, x_2, x_3)\) of non-negative integers such that

\[x_1 + x_2 + x_3 \leq 16.\]