HW 2

18.314

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P1. We need permutations which alternate between even and odd numbers. We consider two cases, when n is odd and when n is even. When n is even, we can choose whether the first term is odd or even, so we obtain $2 \cdot \left(\frac{n}{2}\right)! \cdot \left(\frac{n}{2}\right)!$.

When n is odd, the first number is odd, and then there are $\left(\frac{n+1}{2}\right)! \cdot \left(\frac{n-1}{2}\right)!$ choices.

P2. She can first select which couples to use, and then which among the two members of each couple to talk. Hence, there are $\sum_{i=1}^{n} \binom{n}{i} 2^i = 3^n - 1$ ways to do it.

P3. Note that we can take every element of $[n]$, one element at a time, and decide whether it belongs to $A - B$, to $B - A$, to $A \cap B$, to $C - (A \cup B)$, or to none of them. There are 5^n ways to do this. If $A \cap B = \emptyset$, then there are 4^n ways to do it. Hence, there are $5^n - 4^n$ ways according to the problem.

P4.The number is equal to the number of paths that touch $(3, 3)$ minus the number which touch both $(3, 3)$ and $(5, 5)$. The answer is $\binom{6}{3}\binom{14}{7} - \binom{6}{3}\binom{4}{2}\binom{10}{5}$.

P5. Consider the scenarion of Problem 2. This time, the host wants to select a set of n people with a leader who is a girl and who speaks for all of them. This is counted by the LHS immediately, by first selecting the girl and then the remaining part of the set.

On the other hand, for every k, the host can select k girls, $n - k$ guys, and then a leader among the k girls. This gives the RHS.

P6. We use the fact that $\binom{n}{i} = \binom{n-1}{i} + \binom{n-1}{i-1}$ for $i > 0$. If we substitute this on the LHS for $i > 0$ there is a telescoping sum whose only term not getting cancelled is precisely $\binom{n-1}{k}(-1)^k$.

P7. The interesting part is right of the decimal point. As $(\sqrt{11}+\sqrt{10})^{2002} + (\sqrt{11}-\sqrt{10})^{2002}$ is integer and $(\sqrt{11}-\sqrt{10})^{2002} < 1/10$, then the first decimal point is 9.