

**18.314: PROBLEM SET 9 OPTIONAL PROBLEMS**

(O1) Let  $G$  be the Petersen graph. Compute  $\kappa(G)$ .

(O2) Use the Matrix-Tree Theorem to find the eigenvalues of the adjacency and Laplacian matrices of  $C_5$  and  $C_6$ .

(O3) Prove the following Lemma from class. Let  $M_0(G)$  be the reduced incidence matrix of  $G$ , and  $S$  a set of  $p - 1$  edges of  $G$  (where  $p$  is the number of vertices of  $G$ ). Then

$$\det M_0[S] = \begin{cases} 0, & \text{if } S \text{ does not form the set of edges of a spanning tree of } G \\ \pm 1, & \text{otherwise.} \end{cases}$$