

18.314: PROBLEM SET 9 ADDITIONAL PROBLEMS

Problem Set 9: 10.39, 10.45 and the two problems below; due Tuesday Nov 26

(A1) The *complete bipartite graph* K_{rs} has vertex set $A \cup B$, where $|A| = r$, $|B| = s$ and $A \cap B = \emptyset$. There is an edge between every vertex of A and every vertex of B , so rs edges in all. Let $\mathcal{L} = \mathcal{L}(K_{rs})$ be the Laplacian matrix of K_{rs} .

(a) Find a simple upper bound on $\text{rank}(\mathcal{L} - rI)$. Deduce a lower bound on the number of eigenvalues of \mathcal{L} equal to r .

(b) Assume $r \neq s$, and do the same as (a) for s instead of r .

(c) Find the remaining eigenvalues of \mathcal{L} .

(d) Use the previous parts to compute $\kappa(K_{rs})$.

(e) (optional) Give a combinatorial proof of the formula for $\kappa(K_{rs})$.

(A2) Let G be a regular graph. Suppose the eigenvalues of the adjacency matrix of G are $-2, -2, -2, -2, 1, 1, 1, 1, 4$.

(a) Find the number of vertices, the number of edges, and the degrees of the graph G .

(b) Find the number of spanning trees of G .