

18.314: PROBLEM SET 8 OPTIONAL PROBLEMS

(O1) A *trivalent tree* is a tree in which every vertex has degree 1 or 3. Show that every trivalent tree with m vertices of degree 3 has $m + 2$ vertices of degree 1. In particular, the total number of vertices $2m + 2$ in a trivalent tree is always even.

(O2) Let G be a weighted connected graph in which all edge weights are different. Prove that there is exactly one spanning tree of minimum weight.

(O3) Prove that the Petersen graph (p204 of text) has no cycle of length 7.