18.314: PROBLEM SET 7 OPTIONAL PROBLEMS

(O1) Let G be the graph whose vertices are all of the 3-element subsets of $\{1, 2, ..., 8\}$ (so G has a total of $\binom{8}{3} = 56$ vertices), and whose edges connect two vertices S, T if (and only if) $S \cap T = \emptyset$. For instance, there is an edge between 245 and 137, but no edge between 245 and 147. Does G contain a (closed) Eulerian trail?

(O2) Can a 9-vertex simple graph have the degrees of its vertices equal to $8, 8, 6, 5, 5, 4, 4, 3, 1$? Why or why not?

(O3) Let G be a simple graph with $|V| = 2m$, and $\deg(v) \geq m - 1$ for all $v \in V$. Is it true that G must have a Hamiltonian cycle? (Compare with the theorem of Dirac done in class.)