

**18.314: PROBLEM SET 1**

(1.22) Prove that there exists a positive integer  $n$  of that  $44^n - 1$  is divisible by 7.

(1.29) Let  $T$  be a triangle with angles of 30, 60, and 90 degrees whose hypotenuse is of length 1. We choose ten points inside  $T$  at random. Prove that there will be four points among them that can be covered by a half-circle of radius 0.42.

(1.32) One afternoon, a mathematics library had several visitors. A librarian noticed that it was impossible to find three visitors so that no two of them met in the library that afternoon. Prove that then it was possible to find two moments of time that afternoon so that each visitor was in the library at one of those two moments.

(2.18) Let  $n$  be a positive integer. Prove that it is possible to cut up a cube into  $7n + 1$  smaller cubes.

(2.29) Let  $a_0 = a_1 = 1$ , and let  $a_{n+2} = a_{n+1} + 5a_n$  for  $n \geq 0$ . Prove that  $a_n \leq 3^n$  for all  $n \geq 0$ .

(2.33) Let  $a_1 = 5$ , and let  $a_{n+1} = a_n^2$ . Prove that the last  $n$  digits of  $a_n$  are the same as the last  $n$  digits of  $a_{n+1}$ .