1 Lecture review

1.1 Calculating derivatives

1. The derivative of the function \( y = f(x) \) at the point \( x = a \) is the limit

\[
    f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}.
\]

2. The power rule says

\[
    \frac{d}{dx}(x^r) = rx^{r-1}.
\]

3. The product rule says

\[
    \frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x).
\]

4. The quotient rule says

\[
    \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}.
\]

5. The chain rule says

\[
    \frac{d}{dx}(f(g(x))) = f'(g(x))g'(x).
\]

6. Some miscellaneous derivatives are

\[
    \frac{d}{dx}(e^x) = e^x \quad \frac{d}{dx}(\ln x) = \frac{1}{x} \quad \frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x.
\]

1.2 Meaning of the derivative

1. The slope of the tangent line of the graph of \( y = f(x) \) at \( x = a \) is \( f'(a) \).

2. The equation of the tangent line of the graph of \( y = f(x) \) at \( x = a \) is

\[
    y = f'(a)(x - a) + f(a).
\]
# 2 Problems

1. Find the derivatives of the following functions.

(a) \( f(x) = x^5 - x^3 + x^2 - 1 \)
(b) \( f(x) = (x^2 + 3) \sin x \)
(c) \( f(x) = \frac{\cos x}{e^x + x^2} \)
(d) \( f(x) = \sin(x^3 - 2) \)
(e) \( f(x) = 2^x \)

2. Find the equation of the tangent line for the following functions at the specified points.

(a) \( y = e^x + x - 1, \ x = 0 \)
(b) \( y = \tan(e^x), \ x = \ln \pi \)
(c) \( y = e^{\sqrt{x}}, \ x = 4 \)
(d) \( y = \ln(ln x), \ x = e^e \)
(e) \( y = x^x, \ x = 2 \)

# 3 Answers

1. (a) \( 5x^4 - 3x^2 + 2x \); (b) \( 2x \sin x + (x^2 + 3) \cos x \); (c) \( \frac{-\left(e^x + 2x\right) \cos x - \left(e^x + x^2\right) \sin x}{(e^x + x^2)^2} \); (d) \( 3x^2 \cos(x^3 - 2) \); (e) \( (\ln 2)^2 x \)

2. (a) \( y = 2x \); (b) \( y = \pi(x - \ln \pi) \); (c) \( y = (e^2/4)x \); (d) \( y = e^{-(e+1)}(x-e^e)+1 \); (e) \( y = 4(\ln 2+1)(x-2)+4 \)