# 18.787. Topics in Number Theory (Fall 2012)

- Instructor: Sug Woo Shin (`swshin@mit.edu`)
- Class hours: TR 1-2:30
- Classroom: 2-151
- Office: 2-167
- Office hours: Mon 10-11, Thu 2:30-3:30, or by appointment
- Homework: None
- Exams: None
- Grade: Undergrads will be asked to write term papers to receive grades.

**What to expect** The following are sample topics to be covered. Certainly it's too much - I will cut out some of them as we go along.

- Basics on group schemes, abelian varieties, $p$-divisible groups
- CM abelian varieties
- Dieudonné theory (classification of $p$-div. groups over $\overline{\mathbb{F}}_p$ up to isogeny)
- Honda-Tate theory (classification of abel. var. over $\overline{\mathbb{F}}_p$ up to isogeny)
- Serre-Tate theorem/theory on deformations of $p$-div. groups and abel. var. (see Katz below)
- Grothendieck-Messing theory on deformations of $p$-div. groups
- Galois representations associated with abelian varieties
- Moduli spaces of abelian varieties

**Some useful references**

- Mumford, Abelian Varieties, TIFR (reasonably priced in the online AMS bookstore)
- Bosch, Lütkebohmert and Raynaud, Néron Models, Ergebnisse der Math. 21, Springer-Verlag
  (Find at `http://math.arizona.edu/~cais/scans/`.)
- Shatz, Group Schemes, Formal Groups, and p-Divisible Groups (in "Arithmetic Geometry", Springer)
- Tate, Finite Flat Group Schemes (in "Modular Forms and Fermat's Last Theorem", Springer)
- Tate, $p$-divisible Groups (in "Local Fields" proceeding, Springer)
- Demazure, Lectures on p-Divisible Groups, see
  `http://ncatlab.org/nlab/show/Demazure%2C+lectures+on+p-divisible+groups`
  (Click on "web" to download the book chapter-by-chapter.)
- Tate, Classes d'isogénie des variétés abéliennes sur un corps fini
  (Search and get it on `http://numdam.org`.)
- Katz, Serre-Tate Local Moduli
  `https://web.math.princeton.edu/~nmk/old/serretatelocmod.pdf`
- Messing, The Crystals Associated to Barsotti-Tate Groups: with Applications to Abelian Schemes, LNM 264, Springer-Verlag
- Grothendieck, Groupe de Barsotti-Tate et Cristaux de Dieudonné
  (Find at `http://www.math.jussieu.fr/~leila/grothendieckcircle/barsottitate.pdf`.)

**Prerequisite**

- Reasonable knowledge of general algebra, algebraic number theory, and scheme theory would be needed. It would help if you are familiar with elliptic curves.

**Other remarks**

- This course is designed as a graduate course. An undergraduate with proper background knowledge is welcome to attend my class but keep in mind that you may be required to invest a lot to follow the course.