

18.086, Spring 2016

Introduction

Syllabus, Psets etc.

- Course website: <http://math.mit.edu/~stoonp/18.086/>
- 3 PSets (50% grade)
- Computational project & presentation (50% grade)
- Office hours: TBA

Schedule

- Check updated detailed schedule on the course website
- Rough schedule:

February	Finite differences, stability/accuracy, conservation laws
March	Conservation laws, finite volume, levelset methods
April	Solving large linear (and maybe nonlinear) systems, Newton-Krylov methods etc., Optimization
May	Optimization/minimization problems, N-body problems, project presentations

Course book

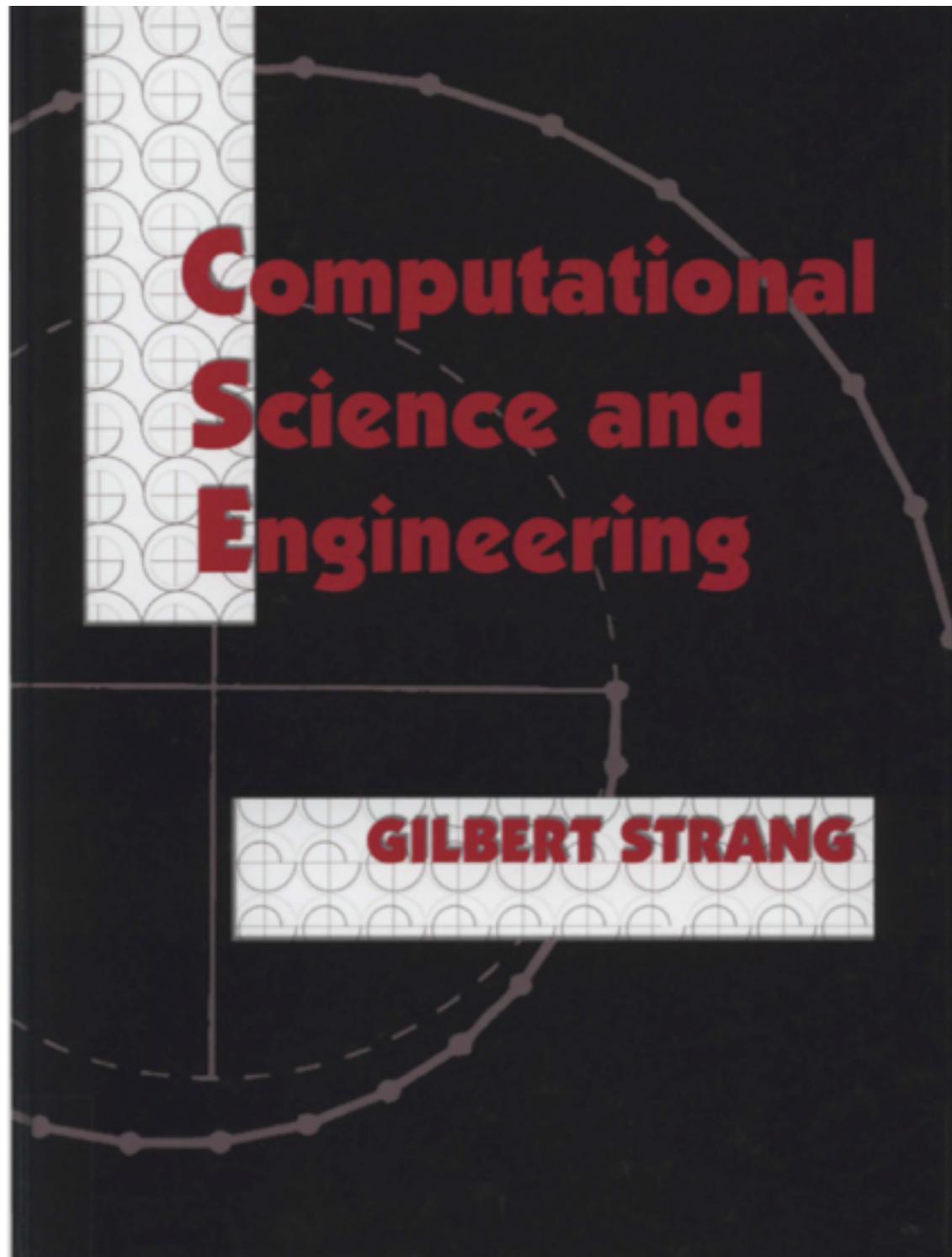


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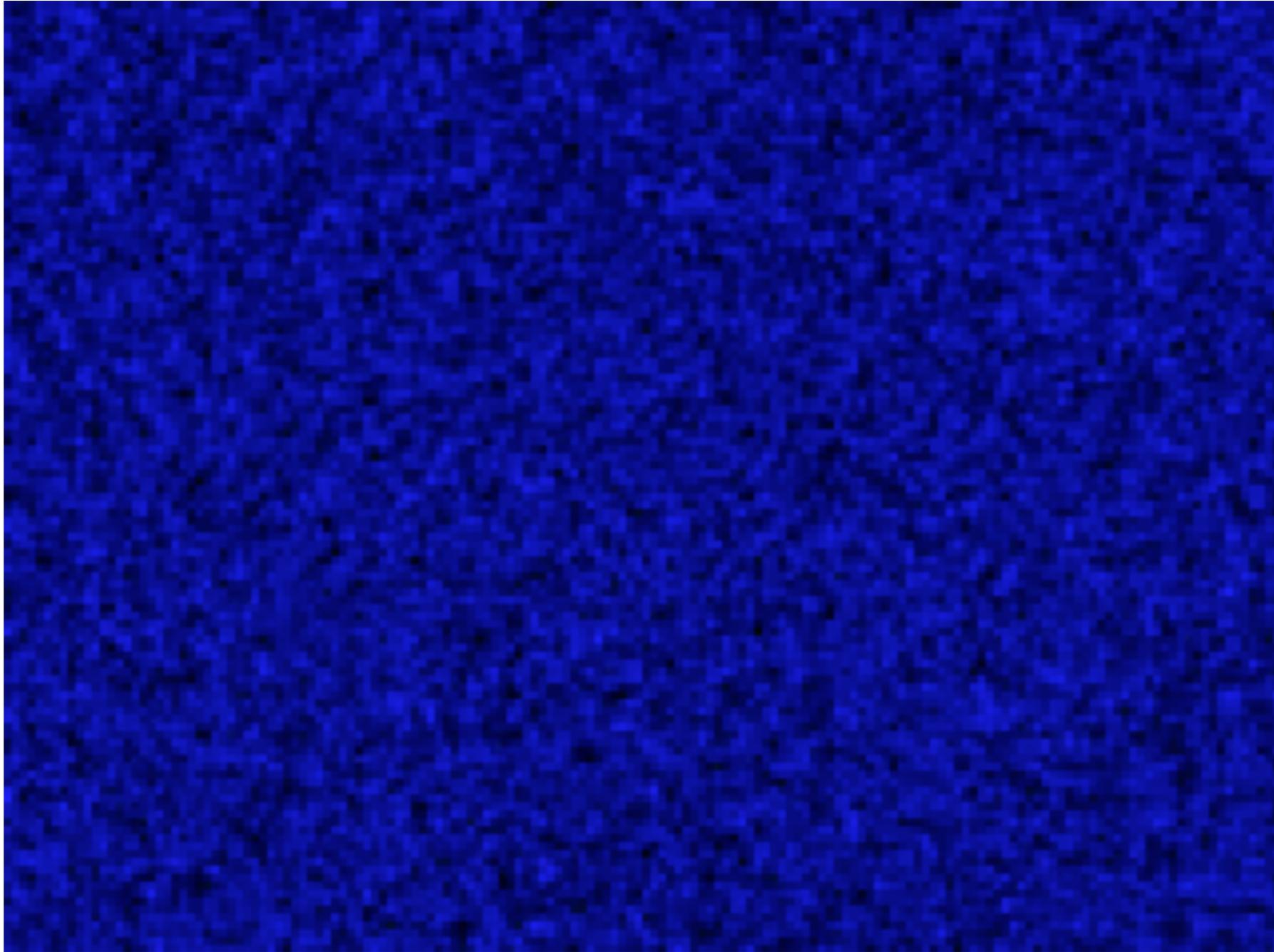
Prerequisites

- Calculus: Gradients/Divergence, Volume/Area/Line integrals, Stokes/Gauss-Theorem
- Linear Algebra: Matrices/Vectors, Eigenvalues and -vectors
- Basic differential equations (ODE/PDE) (18.08)
- Programming: C/C++, Matlab, Mathematica, Python...

Turbulent mixing

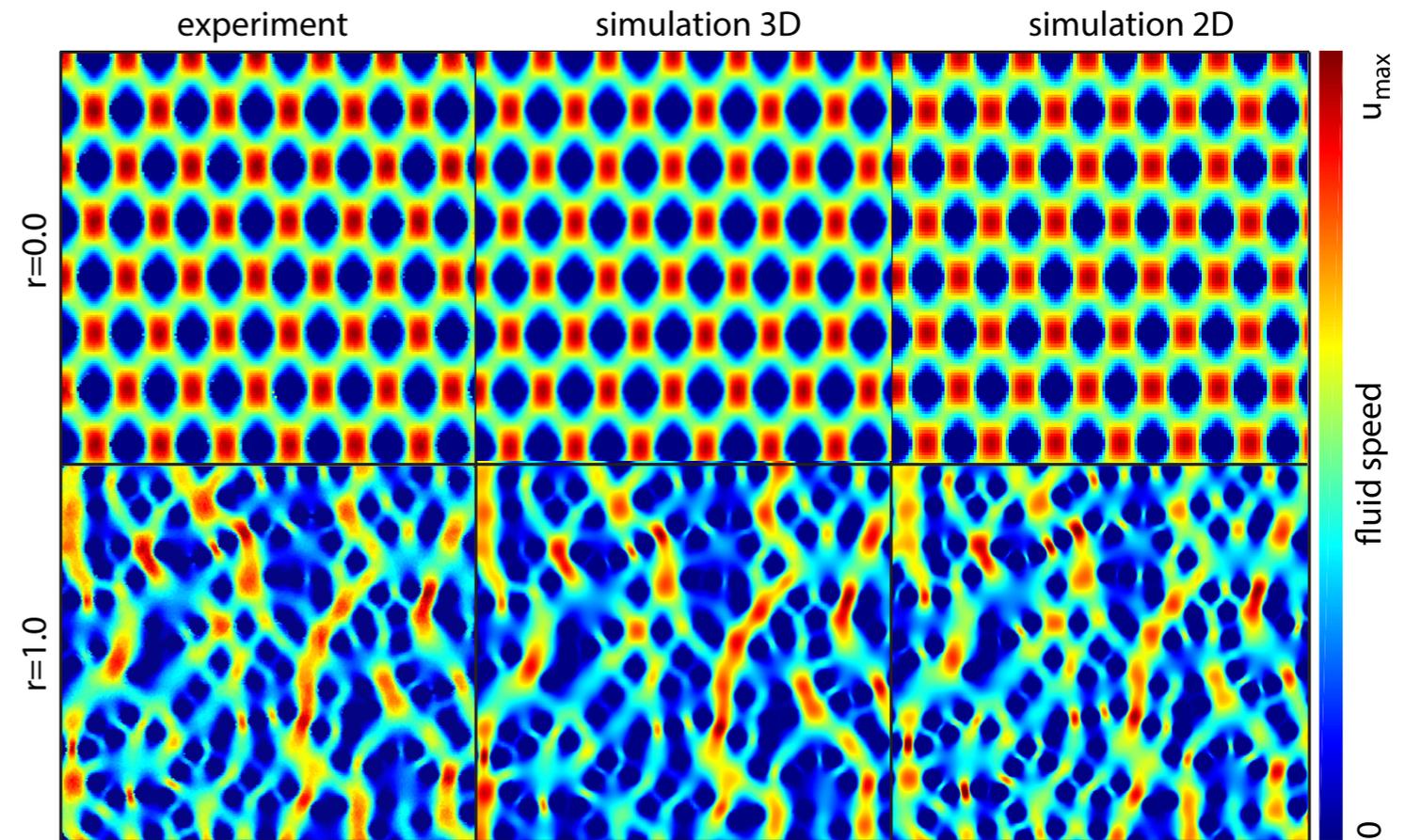
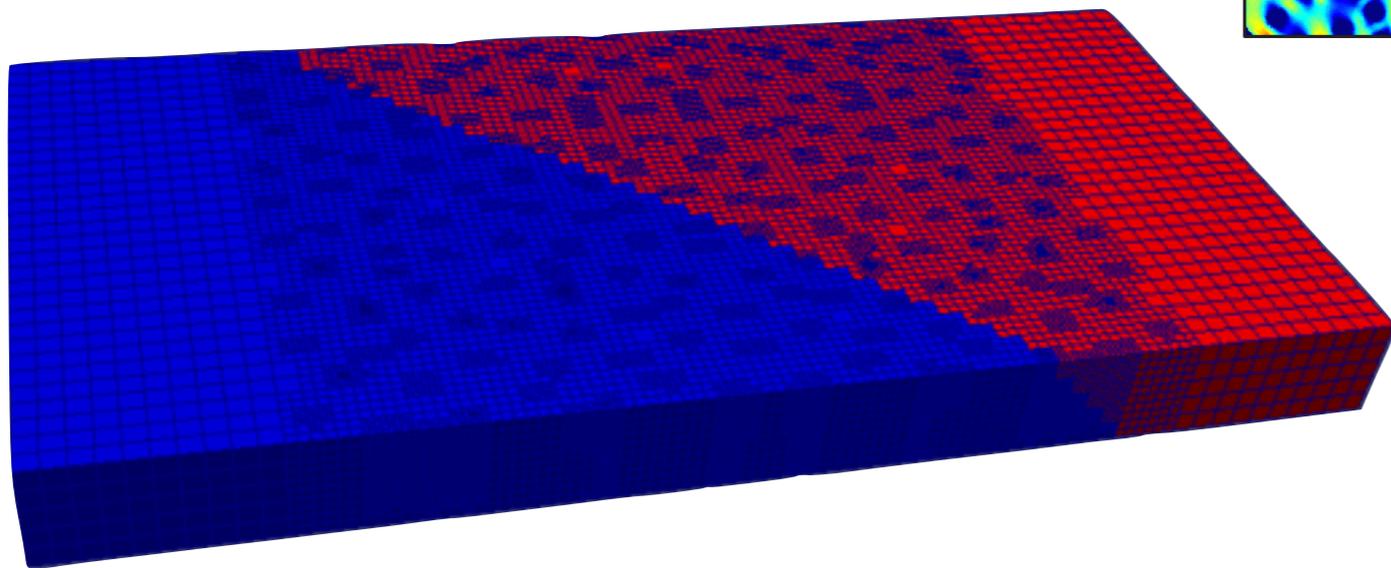
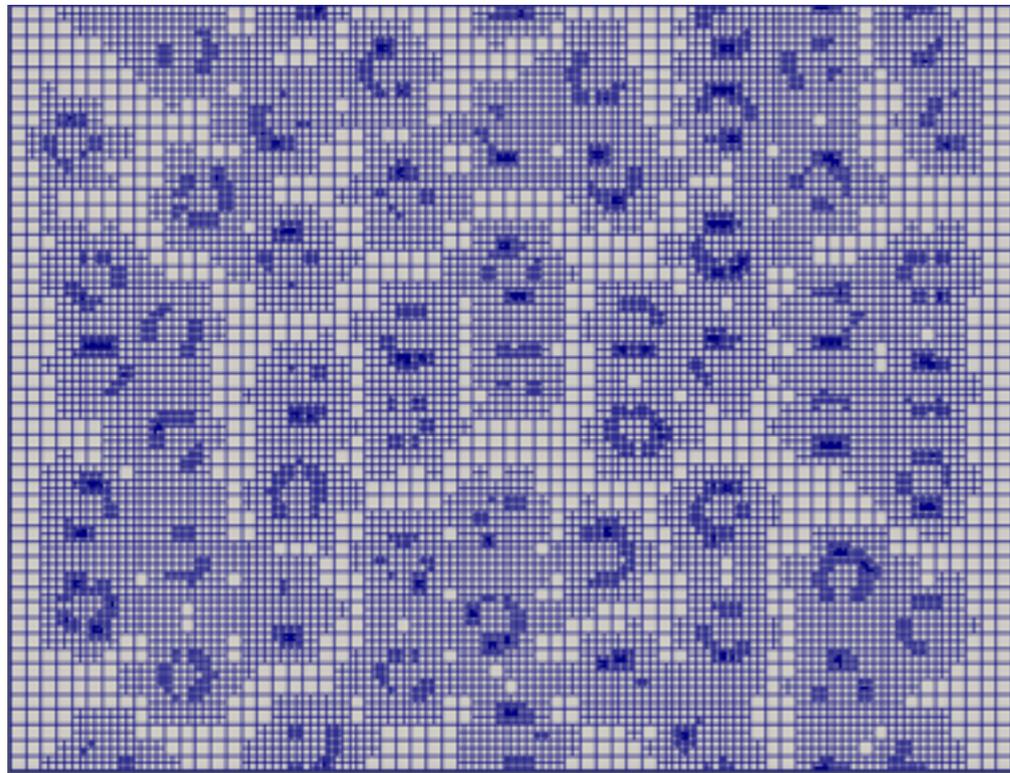
<https://www.youtube.com/watch?v=OM0I2YPVMf8>

Rayleigh-Bernard convection



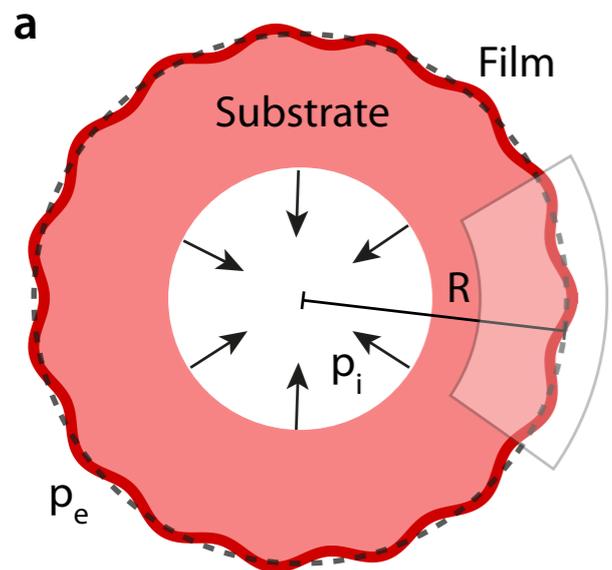
<https://www.youtube.com/watch?v=X5h6hbCxjz8>

Flow in porous media

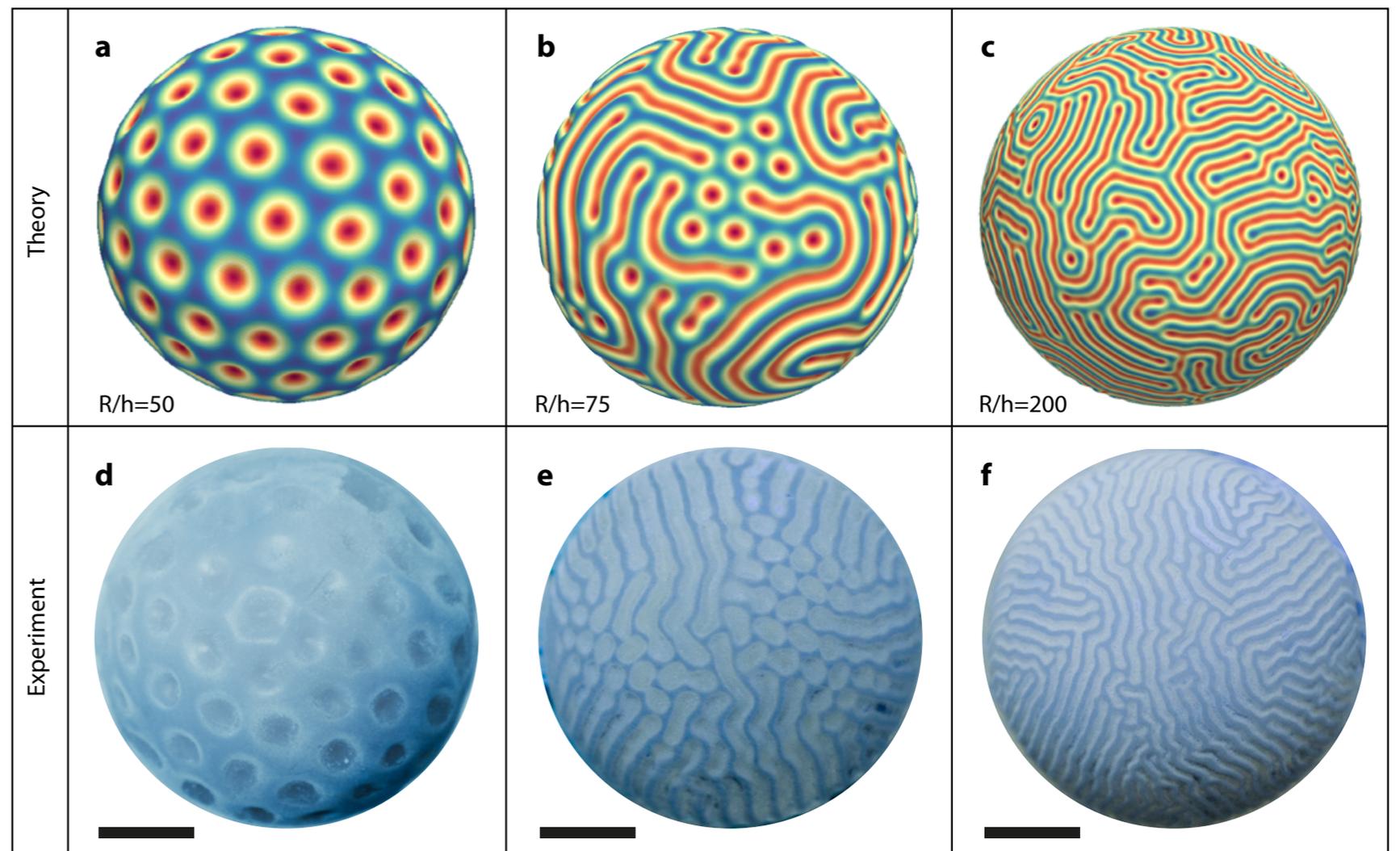


Surface patterns

$$\partial_t u = \gamma_0 \Delta u - \gamma_2 \Delta^2 u - au - bu^2 - cu^3 + (\Gamma_1 + \Gamma_2 u) \cdot [(\nabla u)^2 + 2u \Delta u]$$



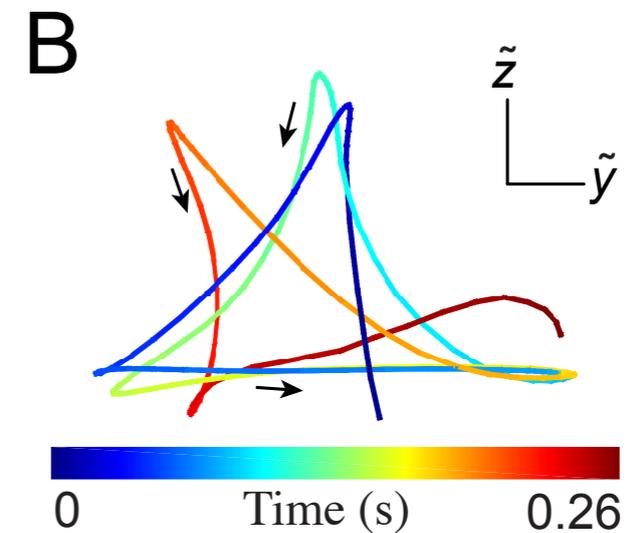
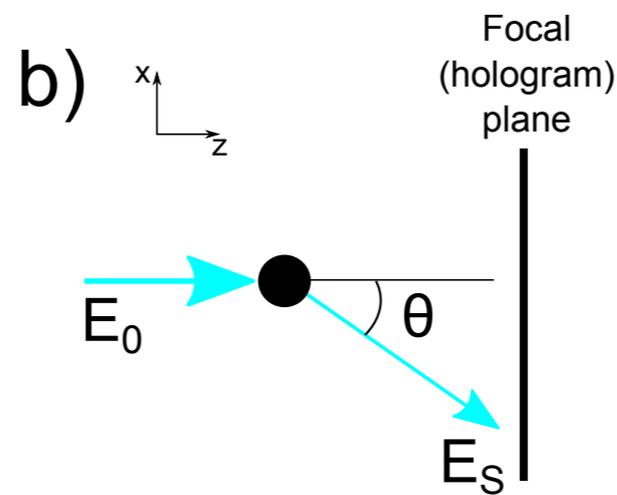
D. Terwange et. al., Adv. Mat. 2014



Increasing effective radius R/h →

3D flagellar beat reconstruction of human sperm cells: **optimization!**

- 2D microscopy:



Propagation of scattered field:

$$h(x', y', z') = \frac{1}{2\pi} \frac{\partial}{\partial z'} \frac{\exp(ikr')}{r'}$$

Where to put scatterers to best “fit” observed scattering pattern? => optimization problem!

